



## **Full Length Article**

# **An Overview of the Simulation of Wave Breaking Over Submerged Breakwaters**

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## **ABSTRACT**

*Natural phenomena in coastal areas, as well as the human activity, such as the expansion of construction in these areas have led to the frequent destructions and environmental disasters in these areas. Coastal erosion is one of this tangible destruction. Construction of submerged breakwater is one of the novel coastal protection solutions which have been considered recently. Since the flow field and the water depth after passing over the submerged breakwaters are important in order to design these structure more efficiently, many simulations and studies have been conducted experimentally and numerically by many researchers to predict the height of the wave passing over breakwaters and the turbulence of current after these structures. In this paper, it is strived to review these studies and present an evaluation of the methods, deficiencies and approaches which they used in their researches.*

**Keywords:** Submerged Breakwater, wave breaking, Navier-Stokes, Turbulence, Free surface

Received 12.04.2014

Revised 21.05.2014

Accepted 21.06.2014

## **INTRODUCTION**

Coastal protection against erosion has always been one of the major challenges of the coastal engineers. This phenomenon is vitally important in cities which are located near the coastal areas and in the vicinity of ports. In the howling wintry storm condition, severe waves and their currents in near shores bring about the transfer of the large amount of sediments into the deeper zones of the coastline. Therefore, erosion happens on some zones of the beach and sedimentation happens on the rest of the zones.

A logical solution for preventing the occurrence of this phenomenon is to apply proper methods of coastal protection. These methods are divided into two groups: soft protection, hard protection. The base of the soft protection is to nourish the beach with sand i.e. filling the eroded zones of the beach with sand. In this method, the materials which obtained either from the dredging operations of naval channels or removing from the zones where sedimentation have happened, will be transferred to the eroded zones and be consolidated. In hard protection methods, however, the hydraulic conditions of the project area have been altered by construction of some structures in near shore or off shore. In other words, the movement of sediments can be controlled by structures such as detached breakwater, groin, seawall and submerged breakwaters.

As it is mentioned, submerged breakwaters are one type of the structures which are built to protect the coastal areas. Like customary breakwaters, submerged breakwaters are rockfill breakwaters but their top part (crest) is below the still water level. The performance of these breakwaters in beach protection is that high waves which are the main cause of transformation of the sediments will break when they reach the zones near the breakwaters because of depth decrease, bottom friction and increase of the wave steepness parameter. This wave breaking leads to the creation of turbulence on the water level, therefore, the wave will lose most of its energy. Then some lower waves generate and pass over the breakwater. The movement of sedimentation is influenced by the structure in two ways. Firstly, some sediments moving

with wave will deposit. In addition, the capability of sediment transportation after the breakwater will decrease because the height and energy of wave have been significantly decreased.

Furthermore, submerged breakwaters act as a filter that the height of crossing wave does not exceed the specified amount. Thus, without changing the natural landscape of the beach they can produce very suitable conditions for marine activities such as surfing, swimming, fishing and tourism growth. In addition to structural effects on the flow field, the fluid flow also affects subsequently on breakwaters. Thus considerable energy from waves and currents are dissipated in the vicinity of these structures produces turbulent currents. Movements of fluid particles lead to the imposition of tangential forces (the shear forces) and normal (inertia) for granular materials in the environment (including bottom sediments and parts of breakwater) which will result in the displacement and deformation of the breakwaters.

Therefore, the simulation and study of the flow field and the water depth after passing over the submerged breakwaters have been carried out experimentally and numerically by many researchers.

## A REVIEW OF STUDIES

### Comparison of experimental studies and numerical simulations

So far, especially the last two decades, a lot of researches have been conducted to understand the phenomenon of wave refraction in general and wave breaking on submerged breakwater in particular. Due to advancements in the field of experimental methods and techniques, empirical studies have been able to play major role in the understanding of waves breaking. Several researchers, such as Rey (1992), Battjes (1993), Ting (1994), Petti (1994), Ohyama (1994), [5], [16], [4], and Pinto (2005) utilized new experimental methods such as HFA<sup>1</sup>, PIV<sup>2</sup>, LDA<sup>3</sup>, and FLDV<sup>4</sup> in order to measure the velocity field, and pressure field amplitude of the wave breaking on the breakwater. Also they were able to investigate phenomena such as flow separation and the pattern eddy formation around the breakwaters.

As an example, Ohyama (1994) used altimeter devices to measure the height of Stokes waves passing over the submerged breakwater and by studying the fluctuations of the water level in different places, he could examine free component of the waves. Pinto (2005) measured velocity components due to wave breaking over submerged breakwaters in short time steps using LDA method. Then, by time averaging on the time history of the measured velocity field, he was able to separate the velocity field and the instantaneous velocities. Chang [5] measured the flow field while solitary wave was passing over rectangular submerged breakwaters using PIV techniques. Using the obtained velocities, he could also calculate the rotation of field and analyze the generation of turbulence energy.

Although the biggest problem in empirical studies is limitations in experimental measurements, variable climatic conditions and limited access to the site of the case study, restrictions related to the dimensions, accuracy of laboratory equipment, and applying the appropriate physical parameters in laboratory studies are other difficulties of experimental methods. Limitations and problems stated in empirical studies on the one hand, and the improvements achieved by computational methods on the other hand has led to considerable interests on numerical simulation of wave breaking in recent years.

Numerical simulation not only has not the aforementioned restrictions of the experimental studies, but it also gives a more accurate description of the flow and the quantities of turbulence. Numerical modeling based on the Navier-Stokes equations can simulate the flow field with great details. Generation, transmission and dissipation of turbulence can also be clearly simulated using an appropriate turbulence model and add it to the original simulation model. The shape of the free surface at different times can be analyzed using appropriate numerical algorithms to estimate free-surface flows. It can be concluded that the numerical simulation can be used to model wave breaking phenomenon well.

### The numerical views

Limitations that previously existed in the area of computational speed of computers have caused the majority of initial numerical models for simulation of wave breaking many limitations. One of the primary views was the depth averaged models. In these simulations of wave breaking, energy dissipation and breaking time are considered by a series of simple phrases of energy dissipation in the governing equations. The application of these models to simulate a submerged breakwater was used in the studies of Cruz [6] and [15]. In both these studies, a horizontal two-dimensional numerical model has been used to simulate wave breaking of the flow over a submerged breakwaters. Due to limitations in the simulation of wave breaking in these models, the deformation of waves can only be studied. Although the presented method is accurate in terms of the computational view, but, as it was mentioned, it cannot give the details

<sup>1</sup>Hot Film Anemometer

<sup>2</sup>Particle Image Velocimetry

<sup>3</sup>Laser Doppler Anemometry

<sup>4</sup>Fiber Laser Doppler Velocimetry

of the flow field and the turbulence. Also, due to averaging in depth, the variation of vertical velocity of particles cannot be achieved.

In order to simulate the flow field accurately, and to derive the vertical distribution of velocity in breaking waves, other views have been proposed. One of these views is based on Potential Flow Theory. For instance, Boundary Integral Equation Method is one of these views. Gu and Wang (1992) used this equations to simulate wave passing over submerged breakwaters. BIEM method can present the information of the flow well before wave breaking. Before wave breaking, movements of particles except at the bottom and surface boundary layers are non-rotating and therefore potential flow theory is valid [7]. But after wave breaking, the current is completely rotating and therefore the theory and all methods based on it are no longer valid. In order to simulate the turbulent flow field in time of wave breaking, the governing hydrodynamic equations should be analyzed thoroughly. And therefore complete and accurate numerical models are required.

Like all turbulent flows of incompressible Newtonian fluids, breaking waves can also be analyzed by Navier-Stokes equations (for incompressible fluids). Theoretically, direct numerical simulation or DNS, can be used for numerical simulations of wave breaking. But the problem in this type of simulation is that in the case of turbulent flows at high Reynolds numbers since the turbulent fluctuations on very small time scales should be calculated, the increased computational memories are needed and the computation time is too long.

Thus, most of the applications of these methods will be within the lower Reynolds number flows and in a small computational field. Since the wave breaking is always accompanied with high Reynolds numbers, using direct numerical simulations to study wave breaking due to the computational power of modern computers does not seem very convenient and practical.

Another view of numerical simulation of wave breaking is to utilize models based on Time Averaged Navier Stokes or RANS. In these equations, analysis is mostly represented in the averaged movements of particles, and the effects of turbulent fluctuations on the mean flow are considered by the Reynolds stress. To calculate these stresses and turbulence quantities associated turbulence models are used.

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \quad (1)$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right) + \frac{\partial}{\partial x} (2v_x' \frac{\partial U}{\partial x}) + \frac{\partial}{\partial y} [v_x' (\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x})] \quad (2)$$

$$\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right) + \frac{\partial}{\partial y} (2v_y' \frac{\partial V}{\partial y}) + \frac{\partial}{\partial x} [v_y' (\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x})] \quad (3)$$

Many researchers have been used this method to simulate wave breaking in the coastal area, such as [22], [23], [3] and [19]. The Navier-Stokes has been used in order to simulate wave breaking and flow field around submerged breakwaters in various studies. [25] and Huang [18] were investigated passing and breaking of solitary waves over submerged structures using Navier-Stokes equations. Garcia [10] used the model which was proposed by Lin [23] to simulate the passing and breaking of the waves over submerged breakwaters. Hsu [16] investigated the flow field around two sequential submerged breakwaters using the height function technique of for determining the free surface. Shen [28] investigated temporal variations in water level at the time of Stokes waves over submerged breakwaters using RANS equations and k-ε turbulence model.

### Turbulence models

When an external force such as a pressure gradient exerts to a fluid which is moving with average velocity  $V$ , it increases the speed and therefore some changes in momentum of the fluid particles. As a result of this increase of the momentum, fluid particles which move parallel to the mean flow change their path in the direction perpendicular to that direction, and move from a layer to another layer. In fact, the distance that these particles pass in this direction is limited. According to Prandtl's theory its maximum is equal to multiply of a factor, known as Von-Karman friction factor  $k = 0.414$ , to fluid depth ( $y$ ). This random movements of fluid particles cause shear deformation and rotation, and the formation of eddy in the fluid flow. With increasing Reynolds number of flow and with increasing the oscillatory velocity of the fluid particles, the size of the eddy of currents increases. In fact, the Reynolds stresses are function of the speed and size of these eddies which cause energy extraction from the mean flow and its entrance to turbulent movements (eddies), and then its dissipation. This energy, known as Turbulent Kinetic Energy ( $k$ ), is equal to the average of the square of oscillating particles' velocity and with the formation of eddy it comes out of the mean flow, then with the entrance to the eddies, they dissipate with a particular Turbulence Dissipation Rate ( $\epsilon$ ). Set of equations to determine these quantities are used in turbulent flow, are called turbulence models.

In addition to modeling turbulence, other important factors must be considered in numerical simulations of wave breaking. Determination of the position of the free surface at different times, and consideration of the proper boundary conditions for wave generation and propagation in computational domain are the most important cases that are presented in the following comments.

#### **The position of the free surface**

In the analysis of free surface flows, pressure and turbulence fields, time and location the free surface are studied, as well as the velocity fields. In this simulation, the positions and therefore the free surface boundaries at various times change. So, in order to apply appropriate boundary conditions to the free surface, the location of free surface at different times must be completed fully and accurately identified. An important issue in relation to the free surface approximation methods is that approximate methods of the free surface must be able to predict whether any formation such as horizontal, vertical, rotating, falling, etc. in the free surface, independent of the condition variables.

So far, many methods for determining the position of the free surface flow in numerical simulations of the open flows have been studied. Researchers based on different Eulerian and Lagrangian perspectives, have presented various numerical algorithms. These methods are classified into two general categories. First category is known as Linear Interface Method. In these methods, the free surface can be expressed as lines or markers whose details during the calculation are stored in memory. The height function method which was presented by Hirt [23] or Line Segments are some examples of this type of methods. The weakness of these methods is to describe surfaces with complex shapes, such as rotation and fall on its own and therefore cannot simulate the pattern of broken waves. As an example, we can refer to the numerical study of Hsu [17] in which he used the height function method to the study of passing waves over consecutive submerged breakwaters.

There are also other methods such as Region Methods and Volume Methods. These methods are very good at defining surfaces with more than one free boundary. The first method of this type is Marker and Cell method or MAC presented by [29]. In this method, massless marked particles were used to determine the position of free surface. These marked particles move with free surface and specify at any time which computational cell are full, empty or contains the free surface. The problem with this method is to determine the exact position of the free surface (especially in the case where the flow is experiencing a complex surface deformation), require a large number of marked particles. This needs larger memory for data storage and also increases computational time.

Therefore, another method known as Volume of Fluid or VOF method was presented in 1981 by Nicolas and Hirts. It follows the same idea of the previous method, with the difference that in this way the computational cell fraction occupied by the fluid free surface is used as a defining factor. Nowadays, this method is one of the most efficient methods of analysis of free-surface flows, not only in hydraulic issues but also other topics such as the simulation of the movement of the molten in casting Engineering.

#### **Proper boundary conditions for wave generation and propagation**

Despite of determining the position of the free surface at different time and space, another problem which mentioned was the application of proper boundary conditions for wave generation and propagation. Wave formation at input boundaries and absorbing the reflected waves at the same time and in the same boundary is a problem not only in numerical simulations, but researchers are also facing in laboratory studies. Up until now, it has been tried to overcome this problem by different methods. For instance, in laboratory studies, Schaffer (1994), proposed using of an absorber wave generator. In the proposed method, water level in the vicinity of the wave generator was measured consistently and thus, reflected and generated waves can be separated from each other. The wave generator is configured to absorb the reflected wave and therefore the interference between reflected waves and generated ones will be solved. On numerical simulations, different ways have been suggested to fix the problem associated with the interference of reflected and generated waves. For example, on the one-dimensional shallow water equations, Kabayashi (1987) presented approximate boundary conditions for absorption of reflected waves coincide with the wave generation. In this situation, the assumption of linear superposition of reflected waves and generated waves was taken, and therefore the condition was true only in the short-wave range. In addition, Wei and Kirby (1995) showed that in long-term simulations, using the boundary conditions with lower accuracy may lead to noticeable numerical errors.

On the models that are based on using the Navier-Stokes equation, using the boundary condition generation-absorption would also be more difficult. Because in these models, information about the free surface displacement and velocity distribution in the boundary wave generator will be required simultaneously. Thus, the model is very sensitive to numerical errors generated at the boundaries of (compared with the Averaged Depth model). So, applying the wave generation-absorption condition in these models should be done with care and attention. Troch and de Roque (1999) presented a generation-absorption boundary condition for numerical models of VOF type. In this method, the velocity

field is measured at a point inside the computational domain. After the measurement of the velocity field, the act of digital filtering has been applied on the velocity field. Then, a signal is obtained by superposition of the velocity signals which is used to correct the signal of generated waves. So, the wave reflected at the boundary is somehow absorbed and the generated wave is corrected. Although the aforementioned method has good accuracy, it is complex as well.

Petit [26] proposed a type of generation-absorption boundary conditions for waves at input boundaries. In this method by using existing information in the computational cells and marginal cells during the previous computational time domain and theoretical values for the desired wave length, velocity values and positions in surface of water at the inlet boundary was calculated at the new time step. Because of using the multiple rows instead of one row of marginal cells, this method was capable of producing waves with good accuracy in the computational domain. Another advantage of this method is that not only can it produce waves based on theoretical equations of waves, but it is possible to simulate the waves which cannot be described by the theoretical equations if the experimental data concerning the time history of the velocity and the free surface are available.

In addition to generating wave at inlet boundary, imposing conditions that cause the exiting of the wave from the end of computational domain and without reflecting into it are also one of the issues that must be considered in the conditions of producing waves. In most numerical simulations, the wave dissipation over a relatively long spongy region (about 2-3 times the wavelength) in the vicinity of the ending borders, has been done using a damping coefficient. For example, simulations carried out by Soliman (2003) can be noted. Although the method has a good computational accuracy, due to excessive computations in spongy region, it is not affordable computationally.

Another common method is using of Open Boundary Conditions or Radiating Boundary Condition at the output boundary has been employed by many researchers. In this method, velocity values at boundary and free surface have been calculated based on the information of adjacent cells and by solving of a differential equation of wave transmission, and then they are applied in the boundary cells.

Except for the above methods which embrace production of the desired wave and the reflected wave at the simultaneous absorption at input boundary, some relatively new methods for wave generation has been used recently. This method which was presented by Lin and Liu [24], a mathematical function of generator of the internal wave is being used. This function is calculated based on the integration of water level fluctuations at a wave period, and it is capable of producing different waves in the computational domain. This method in the range of very shallow waters will produce with relatively high errors.

## CONCLUSION

Submerged breakwaters are one type of the structures which are built to protect the coastal areas. They act as a filter that the height of crossing wave does not exceed the specified amount. Thus, without changing the natural landscape of the beach they can produce very suitable conditions for marine activities. Considerable energy of waves and currents are dissipated in the vicinity of these structures produces turbulent currents. Movements of fluid particles impose some forces to the granular materials in the environment (including bottom sediments and parts of breakwater) which will result in the displacement and deformation of the breakwaters. Therefore, the simulation and study of the flow field and the water depth after passing over the submerged breakwaters are significant to protect the coastline from being eroded.

With considerable improvements achieved by computational methods and limitations of experimental studies, numerical simulation of wave breaking has been acquired great interest recently. Numerical modeling based on the Navier-Stokes equations can simulate the flow field with great details. Generation, transmission and dissipation of turbulence can also be clearly simulated using an appropriate turbulence model and add it to the original simulation model.

In addition to modeling turbulence, other important factors must be considered in numerical simulations of wave breaking. Determination of the position of the free surface at different times, and applying the proper boundary conditions for wave generation and propagation in computational domain are the most important.

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