Bulletin of Environment, Pharmacology and Life Sciences
Bull. Env.Pharmacol. Life Sci., Vol 4 [Spl issue 1] 2015: 457-467

Online ISSN 2277-1808
Journal's URL:http://www.bepls.com
CODEN: BEPLAD
Global Impact Factor 0.533
Universal Impact Factor 0.9804

FULL LENGTH ARTICLE **OPEN ACCESS**

Flow Measurement in Rectangular Channels

Seyed Vahid Nabavi[1]
Department of Civil Engineering, Khomein Branch, Islamic Azad University, Khomein, Iran.
nabavi@iaukhomein.ac.ir, vahidnbv@yahoo.com

ABSTRACT

Open channel flow measurement structures are so designed that the flow rate can be reliably determined from measurement of the upstream head relative to a reference level in the 'control section' of the structure. To measure flow discharge and study the erosion at the brink of free overfalls, the computation of the end depth relationship (EDR) is required in civil engineering practices. A free overfall at the end of an open channel provides a simple means for measuring flow discharge. In fact the discharge can easily be estimated if the critical depth of flow is known and there is a correlation between the brink depth and the critical depth of flow. Due to its practical importance, since Rouse (1936) which was probably one of the first to examine the EDR many investigators studied the free overfall in various channels. This paper presents important laboratory experimental and theoretical investigations on rectangular free overfall.

Keywords:Brink depth; End Depth Ratio; End Depth Discharge, Flow measurement, Rectangular channel

INTRODUCTION

When the water available from a particular source is limited and must be used very carefully, it is useful, and even necessary, to measure the discharge at various points in the system and the flow at farmers' intakes. Also, where farmers have to pay for the water used, discharges should be measured. Flow measurements may also be useful for settling any disputes about the distribution of the water. In addition, measurement of the flows can provide important information about the functioning of the irrigation system. Canal discharges can be measured without structures. The overfall refers to the downstream portion of a rectangular channel, horizontal or sloping, terminating abruptly at its lower end. If it is not submerged by the tail water, it is referred to as the free overfall (Fig. 1). A vertical drop of a free overfall is a common feature in both natural and artificial channels. Natural drops are formed by river erosion while drop structures are built in irrigation and drainage channels as energy reducing devices especially where the flow is supercritical. The free overfall is of distinct importance in hydraulic engineering, aside from its close relation to the broad crested weir, for it forms the starting point in computations of the surface curve in non-uniform channel flow in which the discharge spills into an open reservoir at the downstream end. The problem of the free overfall as a discharge measuring device has attracted considerable interest for more than 80 years and the end depth discharge relationship has been extensively studied by carrying out the theoretical and experimental studies at free overfall of a channel in order to establish a relationship between the critical depth, h_c and the brink depth (end depth), h_b. Rouse [1] was the first to recognize the important feature of the free overfall at the end of channels, proposing a term end-depth-ratio ($EDR = h_b/h_c$ = end-depth/ critical-depth). Since the pioneering work of Rouse, free overfall has been studied extensively for the last several decades, primarily through the laboratory experiments and analytical and theoretical study. However, the problem still provides great challenges to engineers and researchers, as evidenced by the continuous heavy publications. On the other hand, the practical reason for such study is that free overfall can be used as a simple flow measuring device since the flow discharge per unit width q in a rectangular channel can be estimated by $q=[g\ h_b^3\ (h_c/h_b)^3]^{1/2}$ if the end water depth ratio (EDR) can be determined (Guo et al. [2]). In order to find the EDR for different channel conditions, many studies have been carried out on smooth and rough channels and most of them are shown by Dey [3] and Swetapadma et al. [4]. This paper presents some publication which has been investigated effect of bed roughness and slope on the flow over rectangular free overfall.

1-Corresponding Author

Fig. 1. (a) Snapshot of developing flow over a drop; (b) flow over a free overfall, and (c) free surface profile

Hydraulics of free overfall

If the flow at an abrupt end of a long channel is not submerged by the tail water, it can be referred to as a free overfall. In channels with mild slopes, the approaching flow is sub-critical (Fig. 2). At the upstream control section with critical depth h_c, vertical accelerations are negligible and a hydrostatic pressure distribution can be safely assumed. At the brink section with depth h_b the pressure distribution is no longer hydrostatic both due to the curvature of the flow and the aeration of the under nappe. Since there is a unique relationship between the critical depth and flow discharge, the ratio of the end- depth to the critical depth ($EDR = h_b/h_c$) offers a possibility to predict the flow discharge and study erosion at the brink of a free overfall. For steep slopes, where the approaching flow is super-critical, flow discharge is a function of end-depth, channel slope, and channel roughness.

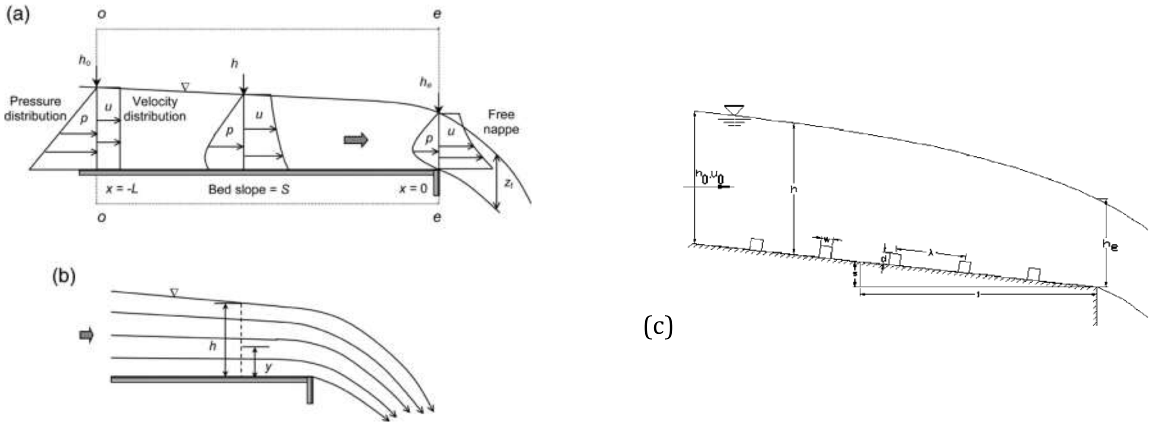

Fig. 1. (a) Schematic view of a typical free overfall; (b) streamline pattern of a free overfall [3]; (c) overfall over rough bed [26]

REVIEW OF LITERATURE [2, 3, 4]

Analytical attempts have been made by many investigators in the past for computation of end depth, among them most of the approaches are based on application of momentum equation with some assumptions which the majority of these studies have been undertaken by Rajaratnam and Muralidhar and few are based on energy consideration and water surface profile at the end section. The numerical solution of two dimensional flows for an ideal fluid has also been attempted adopting various finite element techniques. To obtain three dimensional flow characteristics VOF (Volume of Fluid) model has also been applied. Boussinesq approximation, Free-vortex approach, Weir (without crest) flow approach, Potential flow approach and Empirical approach are some of theory which was used to solve the free overfall theory in channels with different cross sections for both sub & supper critical approaching flow [3].

Bakhmeteff [5] derived Equation (1) which has been the starting point for many attempts to determine the brink depth ratio, (EDR). However, the success of this approach has been limited since the EDR is largely dependent upon cross-sectional shape.

$$E = h + \frac{u^2}{2g} + \frac{u^2 h}{3g}\frac{d^2 h}{dx^2} \quad (1)$$

where E and d^2h/dx^2 is the specific energy and the radius of curvature at the free surface, respectively. Rouse [6] stated that Numerous measurements have shown that this contraction coefficient is equal to the ratio between the crest depth and the computed critical depth, regardless of the rate of discharge or channel width. For supercritical approaching flow he derived Eq. (2), in which $Fr_n^2=q^2/(gh_n^3)$ and h_n are upstream Froud number and flow depth, respectively [3].

$$h_b/h_n=Fr_n^2/(Fr_n^2+0.5) \quad (2)$$

Delleur et al. [7] were one of the first to illustrate the effect of slope on the EDR and studied the variation of end depth ratio (h_b/h_c) using the data of adverse, mild and steep channels. for both smooth and rough surfaces They found that the *EDR* depends only upon relative slope (S_0/S_c), and the bed reoughness had little effect on EDR [3].

$$\frac{h_b/h_c}{2+k_1(h_b/h_c)^3} = \frac{h_n/h_c}{2+(h_n/h_c)^3} = \frac{(S_c/S_o)^{3/10}}{2+(S_c/S_o)^{9/10}} \quad (3)$$

by use of the Manning equation They also reported variation of pressure coefficient (k_1) as a function of relative slope as Table 1.

Table 1- The effect of slope upon k_1 values

Relative slope	k_1
$S_0/S_c< -5$	0.6
$-5 <S_0/S_c<1$	$0.3+1/8\sqrt{1-S_0/S_c}$
$1<S_0/S_c$	0.3

Diskin [8] applied the momentum approach assuming zero end pressure to calculate the EDR value of 0.731 for horizontal rectangular free overfall. Replogle [9] carried out his investigation for rectangular channel based on several assumptions used in Diskin's momentum equation. For rectangular free over fall, he found 0.716 for EDR. Replogle concluded that only the end pressure effectively contributes for the reduction of depth ratio from 1.5 for rectangular over fall to 1.428 as computed by him. The remaining variation from measured value of EDR as 0.715 may be due to inaccurately determination of α values and deviation of actual pressure distribution from parabolic assumption [3].
Markland [10] analysed the problem in terms of the stream function . Employing a finite difference and relaxation technique Markland was able to obtain the flow profile for various Froude numbers. Since Markland used a finite difference method, round off errors were inevitably present. Markland was able to extrapolate a value of 0.720 which compares favorably to Rouse's values of 0.715.
Rajaratnam and Muralidhar [11] applying the momentum approach to a control volume bounded by the brink depth, channel bed, an upstream depth and free surface and neglecting the frictional forces (F), assuming that is small, and taking momentum correction factor=1, For mild, zero and adverse slopes gives equation (4)

$$\rho g\bar{h}_c A_c - k_1 \rho g \bar{h}_b A_b = \rho Q^2(1/A_b - 1/A_c) \quad (4)$$

whereas for steep slopes the expression is:

$$\rho g\bar{h}_n A_n - k_1 \rho g \bar{h}_b A_b = \rho Q^2(1/A_b - 1/A_n) \quad (5)$$

where $\bar{h}$ is the depth of the centre of gravity of the section below the free surface. In order to determine k_1, Rajaratnam and Muralidhar measured the surface profile upstream the end section, (τ) bed shear stress, velocity and pressure distributions. as the upstream distance from the brink increases, the pressure distribution approaches the hydrostatic condition and coincides with it at the critical depth section or normal depth section (for mild slopes or steep slopes, respectively). In supercritical approaching flow, the curve (*EDR* versus S_0/S_c) and (*EDR* versus Fr_n) are shown in Figure 3a and 3b. As it can be seen, The EDR decreases gradually with increase in S_0/S_c up to 1, and then it drops down drastically. Also, The EDR increases with increase in Fr_n, becoming almost constant when Fr_n is greather than 4. The predictions illustrate the correct trend for low Froude numbers, but underestimate the magnitude of the experimental data [3].
In order to ascertain whether the assumption of negligible frictional forces was appropriate, Rajaratnam and Muralidhar measured the centreline bed shear stress using a Preston tube. Using the assumption that

the average shear stress is 10% less than the centreline value, the total frictional forces were calculated from:

$$F = \int_0^L 0.9\tau_0\left(1.5 + 2z\right)dx \qquad (6)$$

where L is the length of the reach under consideration. Rajaratnam et al. stated that individually the shear force term and the weight component could be very large. But they occur in the momentum equation with opposite signs.The analysis thus gives an idea of the error involved when ($W\sin\theta$-F) is neglected in the momentum equation. It can be seen that when S_0/S_c takes a value of 5, it is safe to neglect both terms [3].

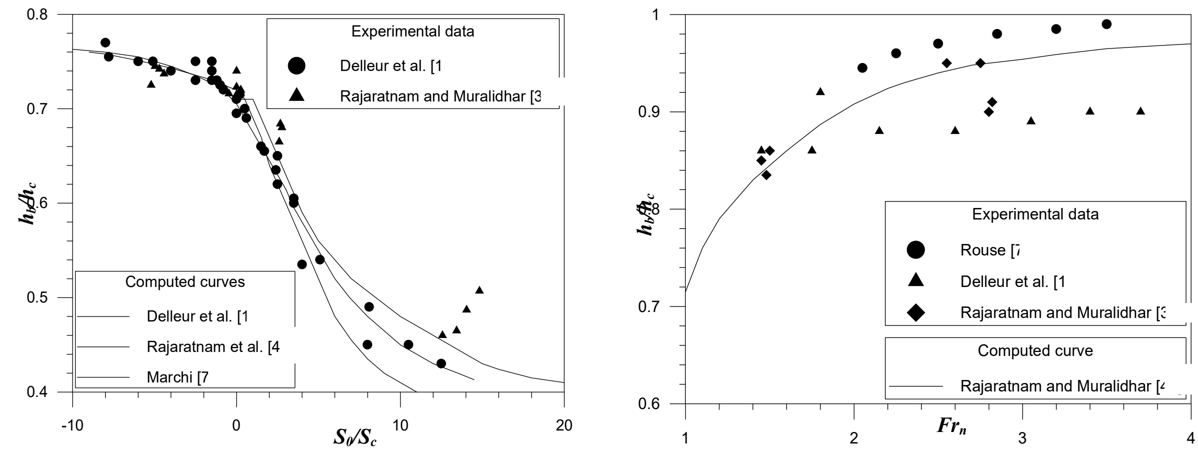

Fig. 3: a) Variation of *EDR*=h_b/h_c versuse relative slope, b) Variation of *EDR*=h_b/h_c versuse Froude number [3]

Anderson [12] used the modified energy equation based on the Boussinesq approximation to obtain an analytical expression for the EDR in rectangular channels. He assumed that beyond the brink the flow is solely influenced by gravity, and derived an expression for the curvature of the water surface. For rectangular channel the obtained equation was,

$$4(h_b/h_c)^3-6(E/h_c)(h_b/h_c)^2+3=0 \qquad (7)$$

for subcritical flow $E=1.5h_c$ and equation (7) reduces to $4EDR^3-9EDR^2+3=0$, which gives EDR = 0.694, (i.e. 3% less as compared to Rouse's classical value of 0.715. in steep slopes Equation (7) can be simple to the Equation (8),

$$E/h_c=(S_c/S_0)^{3/10}+1/[2(S_c/S_0)^{3/5}] \qquad (8)$$

when combined with Manning's equation. Equation (8) when used in conjunction with equation (7) allows prediction of the EDR.

Strelkoff and Moyeri [13] investigated the free over fall at rectangular channel according to potential theory. In this method, boundary value was formulated as an integral equation and then solved numerically. The *EDR* subcritical approaching flow was 0.672. Bauer and Graf [14] experimented rectangular overfall with mild slopes and three types of bed roughness and obtained a constant EDR value of 0.781. However, they pointed out that an insufficient length of the flume might have affected the EDR values. Ali and Sykes [15] applied the free-vortex approach, and estimated the EDR value of 0.678 in horizontal rectangular channels.

Rajaratnam et al. [16] conducted an experimental study with a wider range of roughness and observed that a relative roughness ɛ́ (= k_s/h_c, where k_s is the Nikuradse equivalent sand roughness) less than 0.1, the curve obtained by Delleur et al. [7] was good enough in predicting h_b/h_c, as shown in Fig. 4.

Kraijenhoff and Dommerholt [17] also conducted experiments with slope and roughness. They concluded that the slope or roughness did not change the average value of *EDR* being 0.714. Their value was not significantly affected by either the mild slopes up to 0.0025 or by the bottom roughness, that it was controversial result.

Hager [18] treated two-dimensional free overfall by using the extended energy and momentum equations taking account of the streamline inclination and curvature. The EDR and flow discharge, determined from momentum considerations in the end, is given as Equation (9) and (10), respectively.

$$EDR=9Fr_n^2/(9Fr_n^2+4) \qquad (9)$$

$$Q^2/(gb^2h_b^3)=2/[5j_b^2(1-j_b)] \quad (10)$$

where $j_b=5Fr_n^2/(2+5Fr_n^2)$. For subcritical approaching flow (i.e. $Fr_n=1$), The above equation produces EDR=0.696. The free surface profiles, upstream and downstream the end section, were also investigated by him. Ferro [20] used free overfall as a discharge-measuring structure to establish the relationship between end depth and critical depth. He reported the results of an experimental study on free overfall in horizontal rectangular channels having five different widths. The measurements showed that, for practical application, the relationship between h_b and pressure coefficient (K_1) is independent of channel width. By using this relationship, almost 90% of the estimated discharges are within ±5% of the experimental work. Rai [21] studied the end depth problem and for unconfined nappe; the average value for EDR was 0.712. For relative slope of +5.0, the end depth ratio was found to be 76.14% of end depth ratio of horizontal rectangular channel. The apparent critical section was found to lie at a distance of $3.27h_c$ upstream of end section for horizontal channel and was a function of slope S_0.
Tiwari [22] based on momentum approach and developed computer software, consider Effect of weight of control volume on sloping floor, developed the equation (11)

$$k_1(y_b/y_c)^3-3(y_b/y_c)+2=0 \quad (11)$$

For horizontal channel and zero end pressure, similat to the Diskin, EDR was found to be 0.667.
Bhallamudi [23] has combined Andersen's methods with that of the momentum equation. For the general case of an exponential channel (rectangular, triangular and trapezoidal cross section) he showed that:

$$\left(\frac{2+C_2}{1+C_2}\right)Fr_n^2+\left(\frac{1}{3+C_2}\right)\left(\frac{h_b}{h_n}\right)^{3+2C_2}=\left(\frac{2+C_2}{1+C_2}Fr_n^2+1\right)\left(\frac{h_b}{h_n}\right)^{1+C_2} \quad (12)$$

where C_2 is constant and defined the channel shape. When the upstream flow is subcritical equation (12) may be easily solved. For rectangular channel ($C_2=0$) and When the upstream flow is subcritical ($Fr_n=1$, $h_n=h_c$) equation (12) gives 0.705 for EDR. Bhallamudi's method can also be applied to supercrictial flow with similar results, however the method slightly overestimates the end depth ratio.
Davis et al. [24] presented the results of an experimental study to assess the effect of roughness and slope. It was observed that the EDR is influenced by the slope and roughness. The roughness had more effect at steeper slopes. Two empirical equations were proposed for calculating this relationship, the first requiring only the data of channel slope and the second requiring both channel slope and roughness (n= average Manning roughness coefficient) data. He found that, the first relationship was accurate in predicting 76.7% of the discharges to within 10% and was more useful for estimation of discharge if bed roughness is not known. The second relationship predicted 90% of the discharges to within 10%.

$$EDR=134.84S_0^2-12.66S_0+0.778= 0.8483\exp(-0.2251Fr_n) \quad (13)$$

$$EDR=0.846-0.219(S_0/n)^{0.5} \quad (14)$$

Ferro [34] assumed flow in a rectangular overfall to be similar to that over a sharp-crested weir having a crest height equalling zero. as shown in figure (4a) and (4b) The EDR and discharge estimated from the analysis had a good agreement with the experimental observations of the other investigators. He obtained the following equations for EDR and flow discharge.

$$\left(\frac{h_b}{h_n}\right)=\left(\frac{3Fr_n}{(2+Fr_n^2)^{3/2}-Fr_n^3}\right) \quad (15)$$

$$Q=Fr_n\left(\frac{(2+Fr_n^2)^{3/2}-Fr_n^3}{3Fr_n}\right)^{3/2}Bg^{1/2}h_b^{3/2} \quad (16)$$

for subcritical approaching flow ($Fr_n=1$), Eq. (15) gives the classical result of Rouse $EDR=0.715$, and $Q=1.6542Bg^{1/2}h_b^{3/2}$.

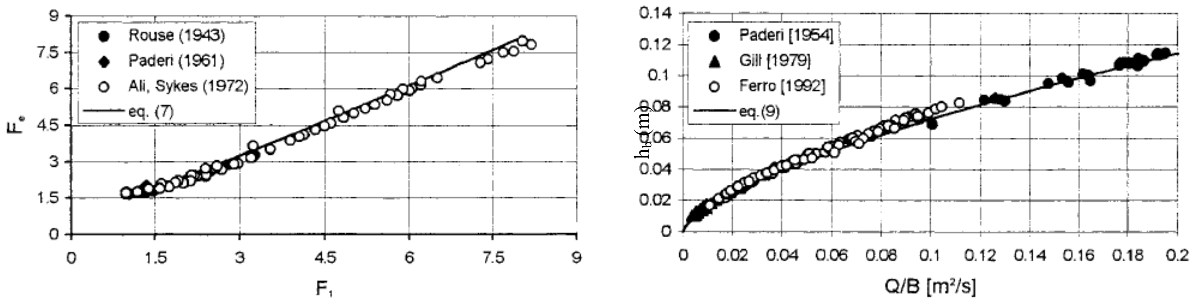

Fig. 4: a) Variation of Fr_b versuse Fr_n, and b) Variation of Q/B versuse h_b [34]

Dey [26] presented theoretical models for free overfall in rough rectangular channels having mild and steep slopes. He applied a momentum equation based on the Boussinesq approximation and the effect of streamline curvature on the free surface was utilized to develop the differential equation of the free surface profile. An auto recursive method was developed to solve the equations simultaneously. Estimation of discharge from end depth and Nikuradse equivalent sand roughness was also done. The computational results agreed satisfactorily with the experimental data of Rajaratnam et al. [16]. The variations of EDR with relative roughness ($\acute{\varepsilon}=k_s/h_c$), and the dependency of $g^{0.5}B^{2.5}/Q$ on $\acute{\varepsilon}_b=k_s/h_b$ in subcritical approaching flow are shown in figure (5a) and (5b), respectively. The EDR decreases with increase in relative roughness $\acute{\varepsilon}=k_s/h_c$. However, the trends of curves of Rajaratnam et al. [16] and Dey [26] are different to some extent. On the other hand, in supercritical approaching flow (steep slope), Fig. (6a) shows the variations of *EDR* with $\acute{\varepsilon}=k_s/h_c$ for different S_0/S_c, and Fig. (6b) presents the dependency of $g^{0.5}B^{2.5}/Q$ on $\acute{\varepsilon}_b=k_s/h_b$ for different S_0/S_c. The value of *EDR* and $g^{0.5}B^{2.5}/Q$ decreases with increase in $\acute{\varepsilon}=k_s/h_c$ and $\acute{\varepsilon}_b=k_s/h_b$, respectively [3, 26].

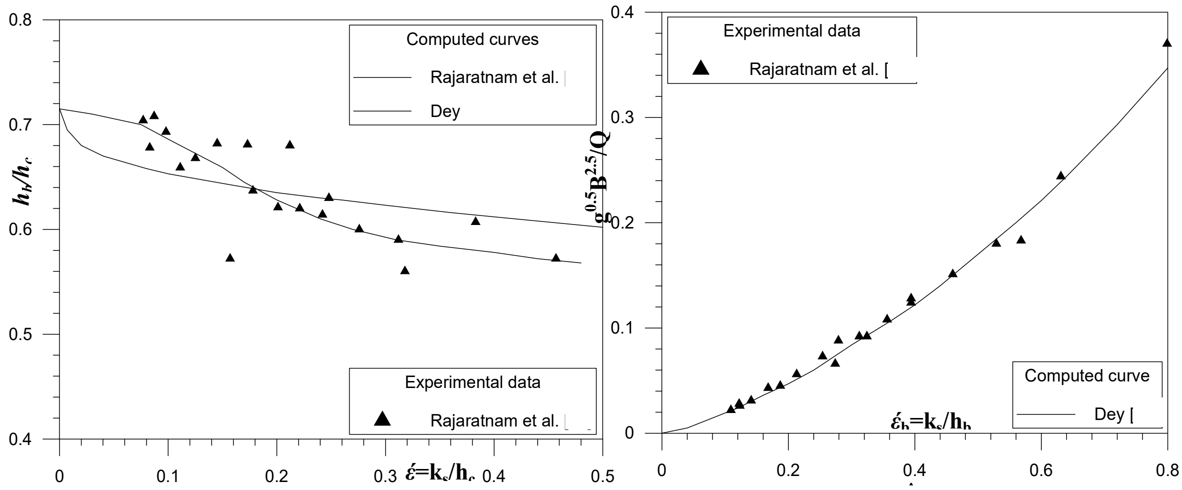

Fig. 5: a) Variation of $EDR=h_b/h_c$ versuse relative roughness ($\acute{\varepsilon}=k_s/h_c$), b) Variation of $g^{0.5}B^{2.5}/Q$ versuse $\acute{\varepsilon}_b=k_s/h_b$[26]

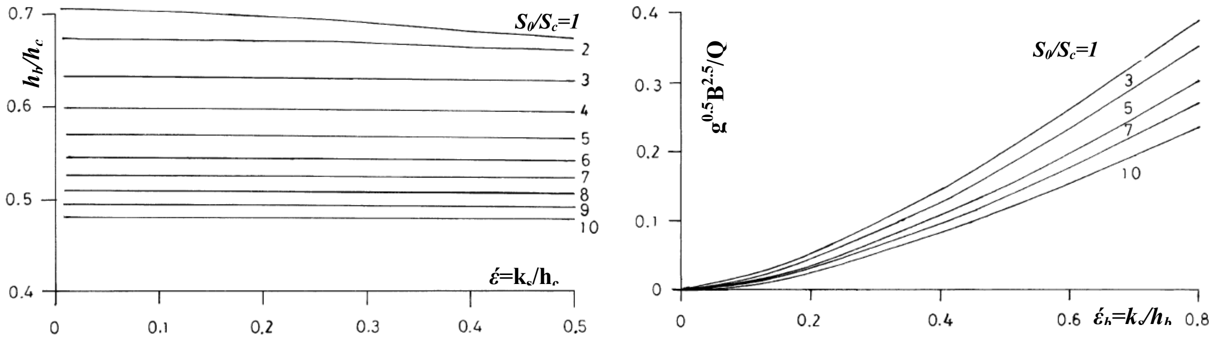

Fig. 6: a) Variation of $EDR=h_b/h_c$ versuse relative roughness ($\acute{\varepsilon}=k_s/h_c$), b) Variation of $g^{0.5}B^{2.5}/Q$ versuse $\acute{\varepsilon}_b=k_s/h_b$[26]

Ahmad [27] developed a quasi theoretical method for determining end depth ratio and end depth discharge (EDD) relationship in terms of pressure coefficient in sub-critical and super-critical flows for rectangular channel. The major equations for EDR and EDD are

$$\left(\frac{h_b}{h_n}\right) = \left(\frac{3Fr_n}{(2[1-C_p]+Fr_n^2)^{3/2}-(Fr_n^2-2C_p)^{3/2}}\right) \tag{17}$$

$$Q = Fr_n\left(\frac{(2[1-C_p]+Fr_n^2)^{3/2}-(Fr_n^2-2C_p)^{3/2}}{3Fr_n}\right)^{3/2} Bg^{1/2}h_b^{3/2} \tag{18}$$

where, C_P is end depth pressure Coefficient and was determined from experimental data. Predicted values of EDR and EDD were compared with experimental data of Ferro [20] and Delleur et al. [7]. For subcritical flows the value of EDR was 0.78 for a confined nappe and 0.758 for an unconfined nappe. For supercritical flows EDR decreases with increase in relative slope and h_c/B. for confined and unconfind nappe, flow discharge were obtained $1.453Bg^{1/2}h_b^{3/2}$ and $1.514Bg^{1/2}h_b^{3/2}$, respectively.

Based on analytical function boundary value theory and substitution variables Guo [28] developed a numerical iterative method for computing free rectangular over fall. The method was applied to calculate water surface profile, pressure distribution and EDR for both smooth and rough channel with a wide range of slope and incoming upstream Froude number. The computed results agree well with the available experimental data. The main advantage of this method was it was very fast and flexible to be applied on curved bed also. The computed results of Guo are shown in Fig. (7a). He indicates that the tendency of the EDR with the bed roughness is similar to that of the experiments (Rajaratnam et al., [16]). That is when the relative roughness k_s/h_c is less than about 0.1, the EDR is almost the same as that of the smooth channel and the channel, can be considered as smooth. When relative roughness k_s/h_c is greater than 0.1 the EDR decreases with the increase of k_s/h_c.

Beirami et al. [29] prepared a theoretical model based on the free vortex theorem and the momentum equation was applied at the brink of free over falls in channels of different cross sections with sub-critical flow. The model was used to calculate the pressure head distribution, the pressure coefficient, the end depth ratio (EDR), and flow discharge at the brink. Their model gave the values of 0.7016 and 0.3033 for EDR and k_1, respectively which had a slight difference 1% to 2% with the values of EDR reported by other investigators.

Guo et al. [2] carried out both experimental and turbulent numerical modeling for the study of free over fall in rectangular channel with strip roughness. A wide range of model parameters like bed roughness, channel slope and upstream Froude number was investigated. The result showed that for a given dimension of bed roughness relative spacing of roughness has a significant effect on the flow. Fig. (7b) shows the variation of EDR versus relative bed slope and some previous work by Delleur et al. [7] and Rajaratnam et al. [16] has been included for comparison. The simulated results are favorably compared with this experimental study as well as the previous work, with the relative maximum error being around 12%. It is seen that the EDR decreases with the increase of relative slope, while the increase of S_0/S_c corresponds to the decrease of roughness or increase of λ/d for otherwise identical conditions. They found that the influence of the strip roughness on EDR decreases as λ/d increases and becomes negligible when $\lambda/d \geq 18$.

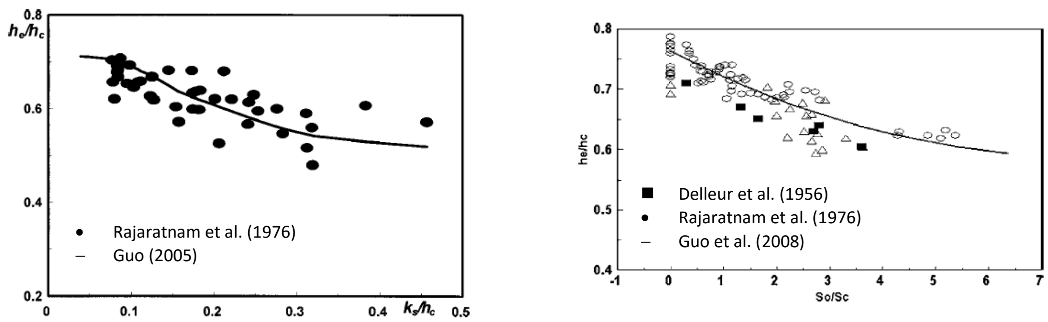

Fig. 7. a) Variation of *EDR* with bed roughness (k_s/h_c) [28]; b) Variation of $EDR=h_b/h_c$ with relative slope S_0/S_c [2]

Tigrek et al. [31] carried out an experimental study n a tilting rectangular flume of 1m width and 12.06m length to get a relationship between brink depth and discharge. in subcritical approaching flow ($Fr_n \leq 1$), the EDR value and discharge per unit width (q) were 0.683 and $5.55h_b^{3/2}$, respectively. The following equation derived for supercritical condition (Fr_n>1). Validity of this explicit discharge brink depth equation was checked and the result agreed well with predicted values.

$$EDR= 0.773-0.018(\sqrt{S_0}/n) \tag{19}$$

$$q=[1/[(0.361-(0.00841(\sqrt{S_0}/n))]]^{3/2}h_b^{3/2} \tag{20}$$

Based on the free vortex theorem the pressure distributions at the brink depth and end-pressure coefficient (k_1) of the free overfall in rectangular channels with sub-critical and super-critical approaching flow theoretically estimated by Nabavi and Beirami [32]. Using the momentum equation, the end depth ratio (*EDR*) was obtained in sub-critical and in super-critical approaching flow. using the Manning's equation flow discharge as a function of end-depth and channel slope was estimated. The experiments were conducted in a stainless steel rectangular flume with one side glass and the other side of glass sides, 400mm in width and 9.5m in length. the Manning's *n* value was 0.011. Experiments were conducted for negative, zero, mild and steep channel condition. For design purposes, charts constructed to facilitate the prediction of flow discharge. They verified their model with available experimental and theoretical results of other investigators. The comparisons showed good agreement with the experimental data. The variation of the h_b/h_c with S_0/S_c and h_b/h_c with the Fr_n are presented in Figs. (8a), (8b). They conclueded, when the relative slope (S_0/S_c) and upstream Froude number (Fr_n) increase, the value of *EDR* decreased. For S_0/S_c =10 and Fr_n=10, the *EDR* reaches to 0.47 and 0.207, respectively. the variation of the h_b/h_n with the Fr_n was presented in Fig. (8c), that can be used for estimated of the flow discharge in super-critical approaching flow.

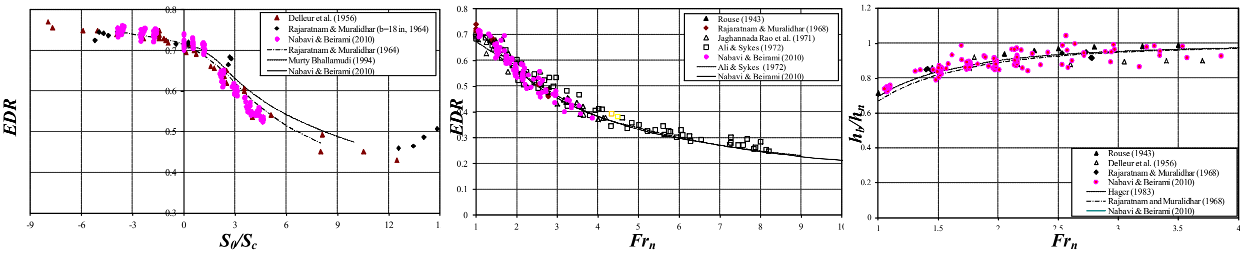

Fig. 8. a) Variation of $EDR=y_b/y_c$ with S_0/S_c, b) Variation of $EDR=y_b/y_c$ with Fr_n c) Variation of y_b/y_n with Fr_n [32]

Mohammed et al. (2011) [33] carried out an experimental study to determine the effect of gravel roughness and channel slope on rectangular free over fall. The experiments were conducted in a metal rectangular flume with glass sides, 300mm in width and 10m in length. The flume was set to slopes of 0, 1/200, 1/100 respectively. For various types of roughness the end depth ratio was expressed as

$$EDR=C_2+C_3[(\lambda/h_c)S_0]^{1/2} \tag{21}$$

C_2 and C_3 were different for different gravel roughness distribution on bed and slope. Six relationships were obtained to predict *EDR*, that all these relationships were compared with Davis et al. [24] and Tigrek et al. [31] to ensure their utility and validity. Table 2 shows the EDR and flow discharhe values obtained by researchers on the rectangular free overfall in sub-critical condition.

Table 2: A brief summary of work undertaken in sub-critical approaching flow

Investigator(s)	*EDR*=h_b/h_c	*Q*	Approach
Rouse (1936)	0.715	$1.654g^{0.5}by_b^{1.5}$	Weir
Diskin (1961)	0.731	$1.6g^{0.5}by_b^{1.5}$	Momentum
Replogle (1962	0.716	$1.6506g^{0.5}by_b^{1.5}$	Momentum
Markland (1965)	0.720	$1.6368g^{0.5}by_b^{1.5}$	Relaxation
Anderson (1967)	0.694	$1.7297g^{0.5}by_b^{1.5}$	Energy
Rajaratnam & Muralidhar (1966)	0.715	$1.654g^{0.5}by_b^{1.5}$	Confined
	0.705	$1.6893g^{0.5}by_b^{1.5}$	Unconfined
Strelkoff & Moayeri (1970)	0.672	$1.8153g^{0.5}by_b^{1.5}$	Potential flow
Bauer & Graf (1971)	0.781	$1.4489g^{0.5}by_b^{1.5}$	Experimentally
Ali & sykes (1972)	0.678	$1.7912g^{0.5}by_b^{1.5}$	Free-vortex
	0.667	$1.8357g^{0.5}by_b^{1.5}$	Momentum
Kraijenhoff & Dommerholt (1977)	0.714	$1.6575g^{0.5}by_b^{1.5}$	Experimentally

Hager (1983)	0.696	$1.7222g^{0.5}by_b{}^{1.5}$	Momentum
	0.692	$1.7372g^{0.5}by_b{}^{1.5}$	Energy
Marchi (1992)	0.706	$1.6857g^{0.5}by_b{}^{1.5}$	Free vortex
Ferro (1992)	0.760	$1.5093g^{0.5}by_b{}^{1.5}$	Momentum
Rai (1993)	0.712	$1.6645g^{0.5}by_b{}^{1.5}$	Unconfined
Murty Bhallamudi (1994)	0.705	$1.6893g^{0.5}by_b{}^{1.5}$	Momentum
Montes (1998)	0.7142	$1.6568g^{0.5}by_b{}^{1.5}$	Potential flow
Ferro (1999)	0.715	$1.6540g^{0.5}by_b{}^{1.5}$	Weir
Guo (2005)	0.712	$1.6645g^{0.5}by_b{}^{1.5}$	Potential
Beirami et al. (2006)	0.7016	$1.7016g^{0.5}by_b{}^{1.5}$	Free vortex
Nabavi & Beirami (2010)	0.717	$1.6471g^{0.5}by_b{}^{1.5}$	Experimentally-Confined
Tigrek et. al (2013)	0.683	$1.7716g^{0.5}by_b{}^{1.5}$	Empirical

Sharifi et al., [35] by applying Genetic Programming a reliable expression of the form of $h_c=Ah_be^{B\sqrt{S0}}$ obtained, where A & B are constants. Firat [22] experimentally studied the characteristics of the subcritical, critical and supercritical flows at the rectangular free overfall to obtain an emprical relation between the brink depth and the flow rate. A series of experiments were conducted by him in a tilting flume with wide range of flow rate and two bed roughness in order to find the relationship between the brink depth, normal depth, channel bed slope and bed roughnesses. Firat proposed an equation to calculate the flow rate if only the brink depth, roughness, and channel bed slope are known. He proposed Eq. (22) for determination of discharge per unit width of rectangular channels,

$$q=\left(\frac{n}{1.63n-0.04\sqrt{S_0}}\right)^{3/2}h_b^{3/2} \tag{22}$$

CONCLUSIONS

The interest of investigators lies in the free overfall owing to a simple means of the flow discharge measurements in open channels. Therefore, an easy to use model like this is attractive. As the computational results are presented in normalized form, their direct application is possible in prototype. For example, in sub-critical approaching flow Table 2, and in super-critical approaching flow Figs. 5b, 6b can be utilized to estimated flow discharge from known end-depth. Till now it has always taken an attention by various researchers and a number of works have carried out in this field. In rectangular channel, many experimental and theoretical works have been carried out to determine a particular relationship between end depth and discharge (i.e. EDR and EDD) for sub critical and super critical approaching flow over smooth and rough beds. it is founded that for sub-critical flow the EDR has a constant value, however, in super-critical flow it depends on relative slope and Manning roughness coeficcient.

The author woud like to acknowledge Islamic Azad University, Khomein Branch for financial support of this research.

Notation

The following symbols are used in this paper:

A = Channel cross section (L^2);
B = Width of rectangular channel (L);
C_1, C_2, C_3 = Coefficient;
d = Roughness height (L);
d^2h/dx^2 = Radius of curvature (L);
E = Specific energy (L);
F = Frictional forces (L);
Fr = Froude number;
g = Gravitational acceleration (LT^{-2});
h = Pressure head (L);
$\bar{h}$= Centre of area of a cross section below the flow surface (L);
h_b= Brink depth (L);
L = Length of control section (L);
n= Average Manning roughness coefficient;
q = Flow discharge per unit width (L^2T^{-1});
Q = Flow discharge (L^3T^{-1});
S = Streamwise slope;
u = Streamwise velocity (LT^{-2});
W = Gravity force of fluid (MLT^{-2});
α = Energy correction coefficient;
β = Boussinesq coefficient;
$\acute{\varepsilon}$= Relative roughness (k_s/h);
λ/d= Relative roughness (*L*);
ρ = Density of fluid (ML^{-3}); and
τ_0 = Shear stress ($ML^{-1}T^{-2}$);

h_c= Critical depth (L);
h_n= Upstream depth (L);
k_1= Pressure Coefficient;
k_s = Nikuradse equivalent sand roughness (L);

Subscripts
b = Brink section;
c = Critical flow; and
n = Far upstream section.

REFERENCES

1. Rouse, H., 1936. Discharge characteristics of the free overfall, Civ. Eng., ASCE, 6(4), 257–260.
2. Guo, Y., Zhang, L., Shen, Y. and Zhang, J., 2008. Modeling study of free overfall in a rectangular channel with strip roughness, Journal of Hydraulic Engineering, ASCE, 134. 664-667.
3. Dey, S., 2002. Free overfall in open channels: State-of-the-art review. Flow Meas. Instrum., 13, 247–264.
4. Swetapadma S., Mittal S.K., and Choudhary. M.K., 2014. Brink Depth at Free Overfall – A Review, International Journal of Engineering Research, 3(10), 594–601.
5. Bakhmeteff, B.A., *Hydraulic of open channels*, McGraw-Hill book company, inc., 1932.
6. Rouse, H., 1943. Discussion on 'Energy Loss at the base of free overfall' by Moore, W.L. Transactions of ASCE, 108(2204), 1383–1387.
7. Delleur, J.W. and Dooge, J.C.I. and Gent, K.W., 1956. Influence of slope and roughness on the free overfall, Journal of Hydraulic Division, ASCE, 82(4), 30–35.
8. Diskin, M.H., 1961. The end depth at a drop in trapezoidal channels, Journal of Hydraulic Division, ASCE, 87(4), 11–32.
9. Replogle, J.A., 1962. Discussion on 'End depth at a drop in trapezoidal channels' by M.H. Diskin, Journal of Hydraulic Division, ASCE, 88(2), 161–165.
10. E. Markland, 1965. Calculation of flow at a free overfall by relaxation method, Proceedings of Institution of Civil Engineers London 31 (May), 71–78.
11. Rajaratnam, N. and Muralidhar, D., 1966. Characteristics of rectangular free overfall, Journal of Hydraulic Research, 6(3), 233–258.
12. Anderson, M.V., Non-uniform flow in front of a free overfall, Acta Polytechnic Scandinavia 42, 1–24, 1967.
13. Strelkoff T., and Moayeri M.S., 1970. Pattern of potential flow in a free overfall. Journal of Hydraulic Division, ASCE, 96(4), 879–901.
14. Bauer, S.W., and Graf, W.H., 1971. Free overfall as flow measuring device, Journal of Irrigation and Drainage Division, ASCE, 97(1), 73–83.
15. Ali, K.H.M. and Sykes, A., 1972. Free-vortex theory applied to free overfall, Journal of Hydraulic Division, ASCE, 98(5), 973–979.
16. Rajaratnam, N., Muralidhar, D., and Beltaos, S., 1976. Roughness effects on rectangular overfall, Journal of Hydraulic Division, ASCE, 102(5), 599–614.
17. Kraijenhoff, D.A. and Dommerholt, A., 1977. Brink depth method in rectangular channel, Irrigation and Drainage Engineering Division, ASCE, 103(2), 171–177.
18. Hager, W.H., 1983. Hydraulics of the plane overfall, Journal of Hydraulic Engineering, ASCE, 109(2), 1683–1697.
19. Marchi, E., 1992. On the free overfall, Journal of Hydraulic Research, 96(6), 259-266.
20. Ferro, V., 1992. Flow measurement with rectangular free overfall, Journal of Irrigation and Drainage Engineering, ASCE, 118(6), 956–970.
21. Rai, A.S., 1993. End depth at a drop in rectangular channels, M.E. Thesis, Civil Engineering Department, Govt. Engg. College, Raipur.
22. Tiwari H., 1994, End depth computations in prismatic channels, M.Tech Thesis, Dept. Of Civil Engg. Manit, Bhopal.
23. Murty Bhallamudi, S., 1994. End depth in trapezoidal and exponential channels, Journal of Hydraulic Research, 32(2), 219–232.
24. Davis, A.C., Ellet, B.G.S. and Jacob, R.P., 1998. Flow measurement in sloping channels with rectangular free overfall, Journal of Hydraulic Engineering, ASCE, 124(7), 760–763.
25. Montes, J.S., 1998. A potential flow solution for the free overfall, Water, Maritime and Energy, Proceedings of Institution of Civil Engineers, London, 130(March), 51-54.
26. Dey, S., 2000. End depth in steeply sloping rough rectangular channels, Sadhana Proceedings of Indian Academy of Science, 25(February), 1–10.
27. Ahmad, Z., 2003. Quasi theoretical end depth discharge relationship for rectangular channels, *Journal Of Irrigation and Drainage Engineering, ASCE, 129(2), 138-141.*
28. Guo, YK., 2005. Numerical modelling of free overfall, Journal of Hydraulic Engineering, ASCE, 131, 134–138.
29. Beirami, M.K. Nabavi, S.V. and Chamani, M.R., 2006. Free overfall in channels with different cross sections and sub-critical flow, Iranian Journal of Science and Technology, 30(B1), 97–105.
30. Firat, C.E., 2004, Effect of roughness on flow measurements in sloping rectangular channels with free overfall, M.E. Thesis, Civil Engineering Department, The Middle East Technical University.
31. Tigrek S., Firat C.E. and Ger A.M., 2008. Use of brink discharge in discharge measurement. Journal of Irrigation and Drainage Engineering, ASCE, 134:89-95.
32. Nabavi S.V., and Beirami, M.K. Flow measurement at the brink of rectangular channels, Proceedings of 9th Hydraulic Conference, Tarbiat Modares university, Tehran, Iran, 2010.
33. Mohammed M. Y., Ahmed Y. Al-taee and Al-Talib Azza N., 2011. Gravel roughness and channel slope effects on rectangular free overfall. Damascus University Journal, 27(1), 47-54.

34. Ferro, V., 1999. Theoretical end-depth-discharge relationship for free overfall, Journal of Irrigation and Drainage Engineering, ASCE, 125(1), 40–44.
35. Sharifi, S., Sterling, M., and Knight, D.W., 2010. Prediction of end-depth ratio in open channels using genetic programming, Journal of Hydroinformatics, 13(1), 36–48.

