



## **Design Z-bend Photonic Crystal Waveguide by FDTD Method**

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### **ABSTRACT**

*In this paper, Z-bend photonic crystal waveguide simulated using the Finite Difference Time Domain (FDTD). To increase the transition in the structure frequency width for incident wave, by creating defects in the basic structure, various structures of the waveguide Z form are designed and then the transition for each other was calculated, compared and the results were analyzed. The results show that the frequency bandwidth can be changed with the changing the cavities position in curvature and also, frequency bandwidth increases as well as some frequencies by adding cavities in the curvature.*

**Keywords:** Photonic crystal, Photonic crystal waveguide, Numerical methods, Finite difference time domain (FDTD), Defect

### **INTRODUCTION**

Nowadays, with the rapid advancement of technology and the manufacture of small electronic components, it is predicted that seldom it will be possible to manufacture smaller parts in upcoming years. Now the analysis of complex electronic circuits is outside of the scope of classical electronics and has been assumed within the scope of quantum mechanics. Thus, the manufactory of devices will be followed in which photons instead of electrons play fundamental role [1].

Currently photonic crystals waveguides are used in engineering and electronics, telecommunications and electro-optics, so it requires extensive research on the use of these tools for different purposes such as use in telecommunications industry, medical devices, and etc. The use of all electronic systems has decreased the speed and the quality of works with some limitations. In this regard, there have been lots of researches conducted on photonic crystals as a good candidate to make photonic elements equivalent with each electronic element, and so design and fabrication of photonic integrated circuits have been carried out [2]. The higher capacity and speed can be reached at the replacement of common electronic elements with photonics elements. Photonics connectors can transfer digital data with capacity of up to three times than that of electronic connectors. Optical signals, unlike the electronic data, can be transferred up to the destination tens of kilometers without attenuation and dispersion. Photonic waveguide and splitters can play a key role in the relationship between the photonic elements and allow photonic crystals integrating in micro-scale and placing many photonic components on a chip. We need high photonics refractive index contrast for a suitable band gap. The radius of curvature in conventional dielectric waveguides is several mm, but the waveguide allows light to bend at the nanoscale [3, 4, 5] because in photonic crystals the light is localized by the periodic structure.

Optical numerical methods based on the division of space may be used for modeling the structure. There are many ways to calculate the properties of photonic crystals that we briefly refer to some of them. Frequency domain methods are preferred when material properties are completely dependent on frequency. In these methods, the calculation must be repeated for each frequency, thus frequency domain methods are suitable for applications that require only a few frequencies. Some methods in frequency domain include: Multiple Multipolar Program (MMP), Finite Element Method (FEM) and Discrete Dipole Approximation (DDA). There is another method of time domain that directly solves Maxwell's equations in the time domains. Time domain methods are applied for systems of nonlinear equations. Another advantage of the time domain method is that they provide device response through one measurement over the entire specified frequency range. Some of the time domain methods are: Transmission Line Matrix (TLM), Finite Difference Time Domain (FDTD), Finite Element Time Domain (FETD) and Finite Volume Time Domain (FVTD).

Finite Difference Time Domain (FDTD) is one of the widely used highly effective time domain methods which straightforward in programming. In this method, both the electric and magnetic fields are calculated without being limited to material properties and is applied for a variety of environments such as dispersive, non-linear and non-homogeneous materials. In this study, we design photonic crystal devices based on coupled waveguides using FDTD method with intention to improve the performance of the devices. First we study photonic crystal and then review waveguide accordingly. Next, we introduce FDTD method and the way we calculate the physical parameters and finally study photonic crystals waveguide Z by applying trial and error procedure and application of photonic crystals key theoretical foundations.

### Photonic crystals

Photonic crystal invention in 1987 was again a different field of science composition [6, 7]. Photonic crystals are based on Maxwell's equations composition for the electromagnetic fields in the dielectric structure and Bloch theory for the solid state physics alternative potentials. The most important aspect in understanding photonic crystal is the understanding of the interaction of these two theories in periodic dielectric structure. The main characteristics of photonic crystals are the regulation of dielectric constant along with the direction of one, two or three in the space. Dispersion relation shows a banned band gap for all energies, that the band gap values are imaginary. Dielectric constant of the medium with a spatial order can be written as follows:

$$\epsilon(\mathbf{r}) = \epsilon(\mathbf{r} + \mathbf{R}) \quad (1)$$

Photon propagation in a magnetic field  $\mathbf{H}(\mathbf{r})$  is obtained using the classical wave equation.

$$\left[ \frac{1}{\epsilon(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) \right] \psi(\mathbf{r}) = \left( \frac{\omega}{c} \right)^2 \mathbf{H}(\mathbf{r}) \quad (2)$$

In this equation, the coordinated mode fields that fluctuate with  $\omega t$  phase factor can be separated. Also, the device modes are determined by the Hermite equation [2]. Generally, an important property of Maxwell's equations and in particular, equation (2) is being independent of scale. This means that if the all dimensions of the device being multiplied at ' $a$ ', answers are the same as before, by the difference that frequencies are divided by ' $a$ '. Due to the scale invariant, it would be appropriate to consider the time and space units dimensionless. A natural length of  $a$  is selected system (rotation period), then all intervals are expressed as a multiple of  $a$  and all angular frequency,  $\omega$  as a multiple of  $2\pi c/a$  [2].

A unique feature of photonic crystals is that they have a banned band gap. The banned band gap almost makes passing electromagnetic waves in a frequency range impossible. The reason for generation of band gap is the difference in dielectric constants. Difference in dielectric constants should increase in order to have a wide band gap. Most of promising applications for two-dimensional and three-dimensional photonic crystals are of photonic band gap position and width [2].

### The photonic crystal waveguide

Among the various devices based on photonics, waveguide play the fundamental role not only on the optical interconnection, but for a variety of angles, fluctuations of channel filters, couplings and switches [8]. High efficacy photoconductivity and chip optical interconnection for telecommunications and optical computing applications are important [9, 10].

Photonic crystal waveguides have better localizations compared to conventional waveguides that work based on the total reflection of light at the junction between the core and shell. Much attention to the potential of photonic crystal waveguide allows form flexible structures in photonic integrated circuits. Photonic crystal waveguide in the design of integrated optical devices components is used in sillier way that of curved waveguide or power isolator with different angles and waste reduction or loss in bending or beam [1].

Light is localized by internal reflection in conventional waveguides such as optical fiber cables. One of the weaknesses of such waveguides is large bandings generated on them, unless the radius of curvature is larger than wavelength, that in this case a large amount of light is wasted. Making large bending radius in optical integrated circuits is difficult; the reason is that there is no space for high bending radius. The key to solving this problem is using a photonic crystal waveguide that can quid light more efficiently in straight or sharp corners; in addition, it is proposed that it can guide light in air that makes less loss due to reduced uptake.

Waveguides are produced by generating the possible linear defects, i.e. the removal of  $N$  rows of cavities in photonic crystals. The waveguide is called  $W_N$  when the  $N$  row is removed. Waveguide with odd  $N_s$  ( $W_1, W_3, \dots$ ) is with symmetrical lines and waveguide with even  $N_s$  ( $W_2, W_4, \dots$ ) is with displaced borders with  $a/2$  with respect to each other. It is better to use waveguides with odd  $N_s$ , because they are symmetric with respect to the axes. Figure 1 shows the  $W_1$  waveguide. In the waveguide  $W_1$ ,  $\omega$  width is consistent with  $a/2$  and in  $W_N$  waveguides is generalized to  $\omega = a\sqrt{3}(N+1)/2$ . Photonic crystal waveguides advantage is that localized the light on the light gap screen. The light cannot radiate in other layers and should follow the bend are be divided into two equal parts in separator, when we generate bend or separator in the waveguide.

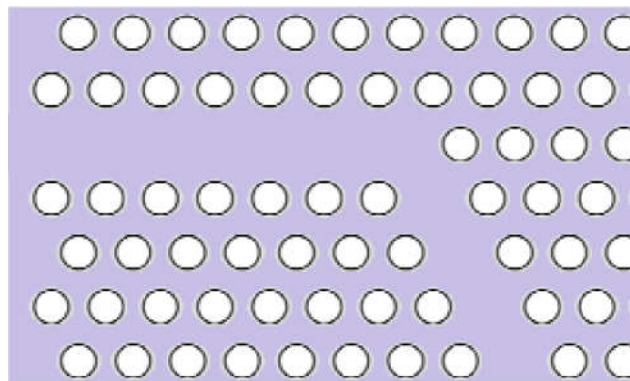


Figure1  $W_1$  waveguide,  $Z$  Photonic crystal

#### Finite Difference Time Domain Method (FDTD)

Finite Difference Time Domain Method (FDTD) is an effective method with increased application on straightforward programming in recent decades. This method solves time-dependent Maxwell equations using second-order differences of space and time and considers both electric and magnetic fields. The main advantages of the FDTD method can be described briefly as follows. Electric and magnetic field s carry evolution in time and do not require saving or converting it to a matrix, do not include any integral equations, Green's functions or singularities, is not limited to the specific material properties and is applied for a variety of environments such as dispersive materials, non-linear and non-homogeneous materials, as well. Device response to a wide range of frequencies at the same time has been calculated and is well suited for parallel programming, and creates a direct grid for a variety of the geometries of finite and infinite periodic structures with arbitrary precision. Finally, enables us to illustrate electromagnetic fields in the environment for the study of the device. FDTD method was successfully performed for optical near-field photonic crystals. In standard FDTD method, the model is divided in cubic cell with curved shapes boundaries. To obtain a very precise result, a very delicate network of model is needed which leads to use of a lot of computer memory and also increased the computational time.

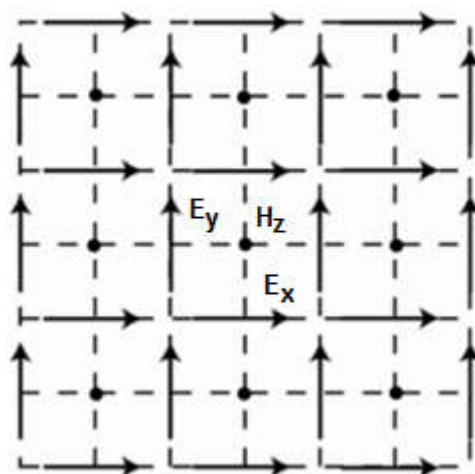


Figure 2 FDTD environment grid simulation and location of electric and magnetic fields on each cell for TE mode

Two-dimensional FDTD simulation is classified into two modes: Transverse Electric mode (TE) with  $E_x$ ,  $E_y$  and  $H_z$  nonzero field components and Transverse Magnetic mode (TM) with  $H_x$ ,  $H_y$  and  $E_z$  nonzero field components. Figure 2 is that of the TE mode, the simulation environment is divided into square cells with field components on each cell on special order. Then, applying the finite difference on time-dependent Maxwell equations, a set of equations for the calculation of various field components are obtained in terms of previous time related values. The equations include:

$$\frac{\partial \mathbf{D}}{\partial t} = \nabla \times \mathbf{H} - \mathbf{J} \quad (3)$$

$$\mathbf{D}(\omega) = \epsilon_0 \cdot \epsilon_r^*(\omega) \cdot \mathbf{E}(\omega) \quad (4)$$

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu_0} \nabla \times \mathbf{E} \quad (5)$$

Normalizing equations (5), (6) and (7) by the relationship:

$$\tilde{\mathbf{E}} = \sqrt{\frac{\epsilon_0}{\mu_0}} \cdot \mathbf{E} \quad (6)$$

$$\tilde{\mathbf{D}} = \sqrt{\frac{1}{\epsilon_0 \mu_0}} \cdot \mathbf{D} \quad (7)$$

The result is:

$$\frac{\partial \tilde{\mathbf{D}}}{\partial t} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \nabla \times \mathbf{H} - \frac{1}{\sqrt{\epsilon_0 \mu_0}} \mathbf{J} \quad (8)$$

$$\tilde{\mathbf{D}}(\omega) = \epsilon_r^*(\omega) \cdot \tilde{\mathbf{E}}(\omega) \quad (9)$$

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\sqrt{\epsilon_0 \mu_0}} \nabla \times \tilde{\mathbf{E}} \quad (10)$$

Here we work with TE mode. In this case, equation (8), (9) and (10) will be as follows:

$$\frac{\partial D_x}{\partial t} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \left( \frac{\partial H_z}{\partial y} - J_x \right) \quad (11)$$

$$\frac{\partial D_y}{\partial t} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \left( -\frac{\partial H_z}{\partial x} - J_y \right) \quad (12)$$

$$D_x = \epsilon_r^*(\omega) \cdot E_x(\omega) \quad (13)$$

$$D_y = \epsilon_r^*(\omega) \cdot E_y(\omega) \quad (14)$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \left( \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \right) \quad (15)$$

$$\frac{D_x|_{i,j+1/2}^{n+1/2} - D_x|_{i,j-1/2}^{n-1/2}}{\Delta t} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \left( \frac{H_z|_{i,j+1}^n - H_z|_{i,j}^n}{\Delta x} - J_x|_{i,j+1/2}^n \right) \quad (16)$$

$$\frac{E_y|_{i+1/2,j}^{n+1/2} - E_y|_{i+1/2,j}^{n-1/2}}{\Delta t} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \left( \frac{H_z|_{i,j}^n - H_z|_{i+1,j}^n}{\Delta y} - J_y|_{i+1/2,j}^n \right) \quad (17)$$

$$\frac{H_x|_{i,j}^{n+1/2} - H_x|_{i,j}^n}{\Delta t} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \left( \frac{E_x|_{i,j+1/2}^{n+1/2} - E_x|_{i,j-1/2}^{n+1/2}}{\Delta y} + \frac{E_y|_{i-1/2,j}^{n+1/2} - E_y|_{i+1/2,j}^{n+1/2}}{\Delta x} \right) \quad (18)$$

In this relations,  $\mathbf{D}$  is electric displacement vector,  $\bar{\mathbf{D}}$  Normalized electric displacement vector,  $\mathbf{E}$  The electric field vector,  $\bar{\mathbf{E}}$  Normalized electric field vector,  $\mathbf{J}$  electric current density vector,  $\mathbf{H}$  the magnetic field vector,  $i$  and  $j$  denotes the cell location, and  $\Delta x$ ,  $\Delta y$  the size of the grid cells,  $\Delta t$  time step and  $n$  time index. In this algorithm, first, the discussed fields are initialized with the appropriate set of initial conditions and then in the time loop new values for each component having adjacent fields in previous steps [12 and 13].

FDTD method constraints for space and time are the size of a step for numerical dispersion space and are stability dependent for time [12]. Numerical dispersion, the value difference between analytical phase and simulation phase velocity, which is inherent in FDTD, is dependent on frequency, network size and the direction of propagation. Numerical dispersion causes nonphysical results solving this problem requires a smaller cell size, so we should find the optimum value for the cell size. To ensure accurate results, the numerical dispersion with the 1.10 cell size wavelength below is recommended [12], namely:

$$\Delta x, \Delta y \leq \frac{\lambda}{10} \quad (19)$$

In the above equation,  $\lambda$  is the shortest wavelength in the simulation range. The lack of stability in numerical simulation makes the calculated results for the magnetic component of the electric field increases without limit. To ensure the stability of the algorithm, the time step should be limited under the below constraint under the title of stability criteria (CFL),.

$$\Delta t \leq \frac{1}{C_{max} \sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}}} \quad (20)$$

In this equation,  $C_{max}$  is the maximum phase velocity of electromagnetic wave in the modeling environment [12].

For the simulation of electromagnetic wave propagation appropriate cut boundary conditions shall be used, which ideally absorbs all output signals regardless of frequency and polarization angle. One of the most effective and most flexible ways of absorbing boundary conditions provided by Berenger [13] is the matching layer (PML). The work is based on that a wave, following the propagation in the A environment to hits the B environment, the intrinsic impedance reflected by the result:

$$\Gamma = \frac{\eta_A - \eta_B}{\eta_A + \eta_B} \quad (21)$$

That is obtained from dielectric constant  $\epsilon$  and the permeability  $\mu$ :

$$\eta = \sqrt{\frac{\mu}{\epsilon}} \quad (22)$$

If we choose  $\epsilon$  and  $\mu$  in a way to remain constant,  $\Gamma$  will become zero and no reflection occurs. The problem still remains because the wave continues to spread in the new environment. We need a dissipative environment so that completely absorb before hitting the boundary. This is done by choosing complex numbers for  $\epsilon$  and  $\mu$  in the equation (22), because the imaginary part causes attenuation [14].

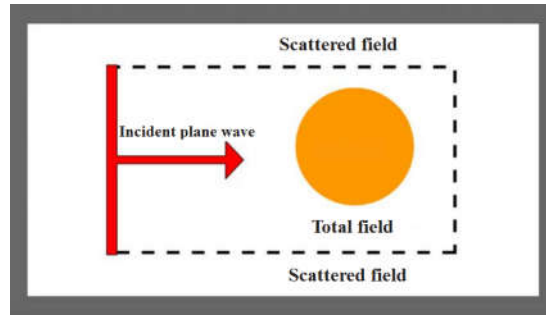


Figure 3 Field incident, Total scattered field, and Corresponding boundaries

Scattered field method-total field is a high efficacy method, especially when the scattering properties are considered. The scattering is determined by putting the flat wave in the computational domain. This method is based on the linearity of Maxwell's equations. The computational domain is divided into two areas: total field and scattered field (Figure 3). The interaction between the incident field and the scattered field occurs in the total field. There is a special relationship in the border between the two areas that incident field is added or removed from of the total field quantities.

### Problem layout in the simulation

In this paper we study the design of two-dimensional photonic crystal waveguide Z. Matrix has a dielectric constant of  $\epsilon = 11.56$  and refractive index of the  $n=3.4$  and air cavities in the matrix are with a radius of  $R=0.29 a$ . Examined grid is triangular with computing dimensions of  $(51 \times 16.454)$ . Investigated modes are TE modes released from a source with frequency of  $0.25 (a/\lambda)$  and pulse width of  $0.1$ . Simulation includes three detectors by means of which we can calculate the reflection and transmission values. Defects are applied in the upper and lower waveguide Z curvature. Figure 4 shows the general layout of the simulation and the location of the defects.

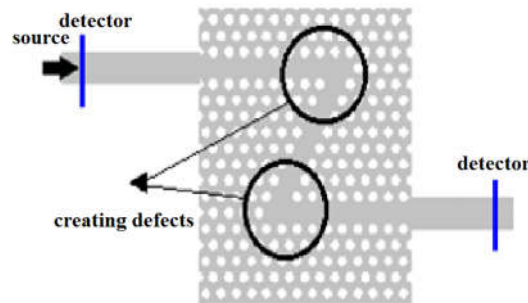


Figure 4 the general layout of the simulation and the location of the defects in photonic crystal waveguide Z

### DISCUSSIONS

We design a photonic crystal waveguide given the simulation layout and using the finite element method described above. Also, due to the fact that always a there is a cavity in the coupling waveguide and curvatures that saves energy and also the waveguide end at the curvature hits the end of the route and reflects upward, we can design and optimize the structure of the coupled waveguides Z. Improve traffic flow is the width of the wave is the most important post simulation stage, so we located a defect in the coupling to increase the two parameters.

Standard photonic crystal waveguide Z is shown in Figure 5 (a). The passage is shown by Ta in Figure 6. We move a cavity in waveguide Z corner of the standard photonic crystal of the curvature to the other side, the changes are shown in figure 5 (b). Also, the passage of this structure is shown in Figure 6 by Tb. In Figure 5 (a) we generate 6 defects with cavity radius of  $0.14$  with three cavities upward and three downwards, Figure 5 (c). The passage value by Tc is shown in Figure 6. We generate 10 defects with cavity radius of  $0.14$  in Figure 5 (b), and as shown in Figure 5 (d) is shown, five defects on upper curvature and five on bottom curvature. The passage by Td is shown in Figure 6. By changing the radius of the cavities in Fig 5 (c) we get that of in Fig. 5 (e), where the cavities radius are from left to right and

from top to bottom are: a 0.0598, a 0.1511, a 0.0598, a 0.0598, a 0.1511 and a 0.0598, and the passage by  $T_e$  is also plotted in Figure 6. we change the radius of the cavities from left to right and from top to bottom in Figure 5 (d), the radius of the cavity are: a 0.1093, a 0.1093, a 0.1775, a 0.1093, a 0.1093, a 0.1093, a 0.1093, a 0.1775, a 0.1093, a 0.1093 (Fig. 5 (f)), the passage by  $T_f$  is depicted in Figure 6. We add four defects to the structure In Figure 5 (g), 2 defects in the top curvature and 2 defects in the bottom curvature, Cavity radius is considered equal a 0.16227, the password by  $T_g$  for this structure is shown in Figure 6. Latest situation is shown in Figure 5 (h). we generate 24 defects in the crystal structure, as before the cavities radius from left to right and from top to bottom are, respectively: a 0.1037, a 0.1295, a 0.2579, a 0.1580, a 0.1295, a 0.1037, a 0.1037, a 0.1295, a 0.1580, a 0.2579, a 0.1295 and a 0.1037 (Fig. 5 (f)), the passage of this structure by  $T_h$  is shown in Figure 6.

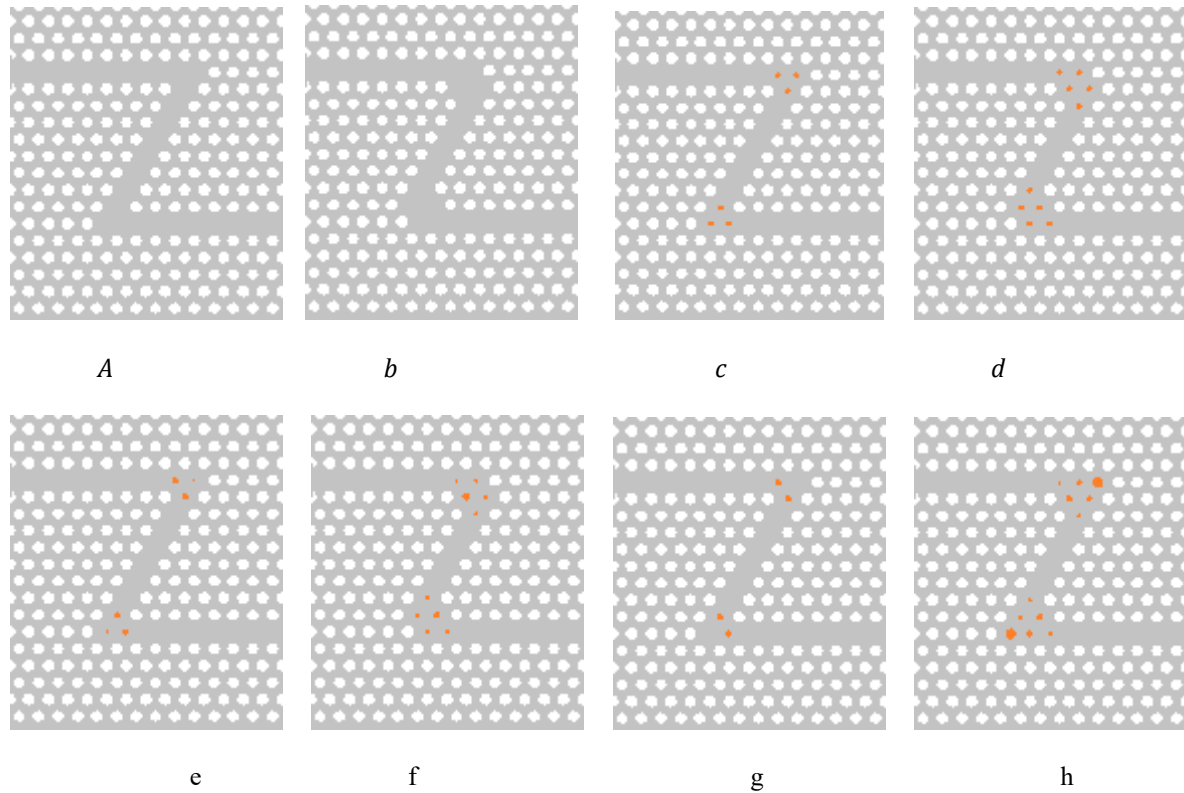
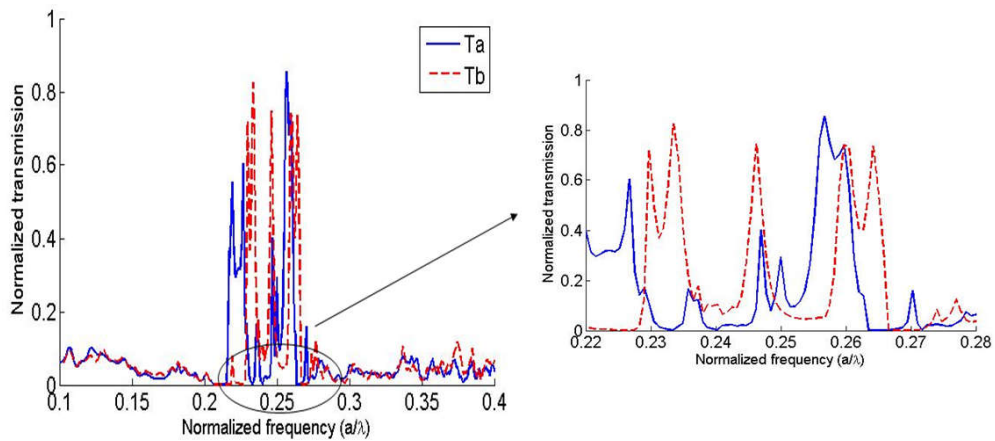


Figure 5 (a) standard photonic crystal waveguide Z, ((b), (c), (d), (e), (f), (g) and (h)) waveguide Z photonic crystals changed



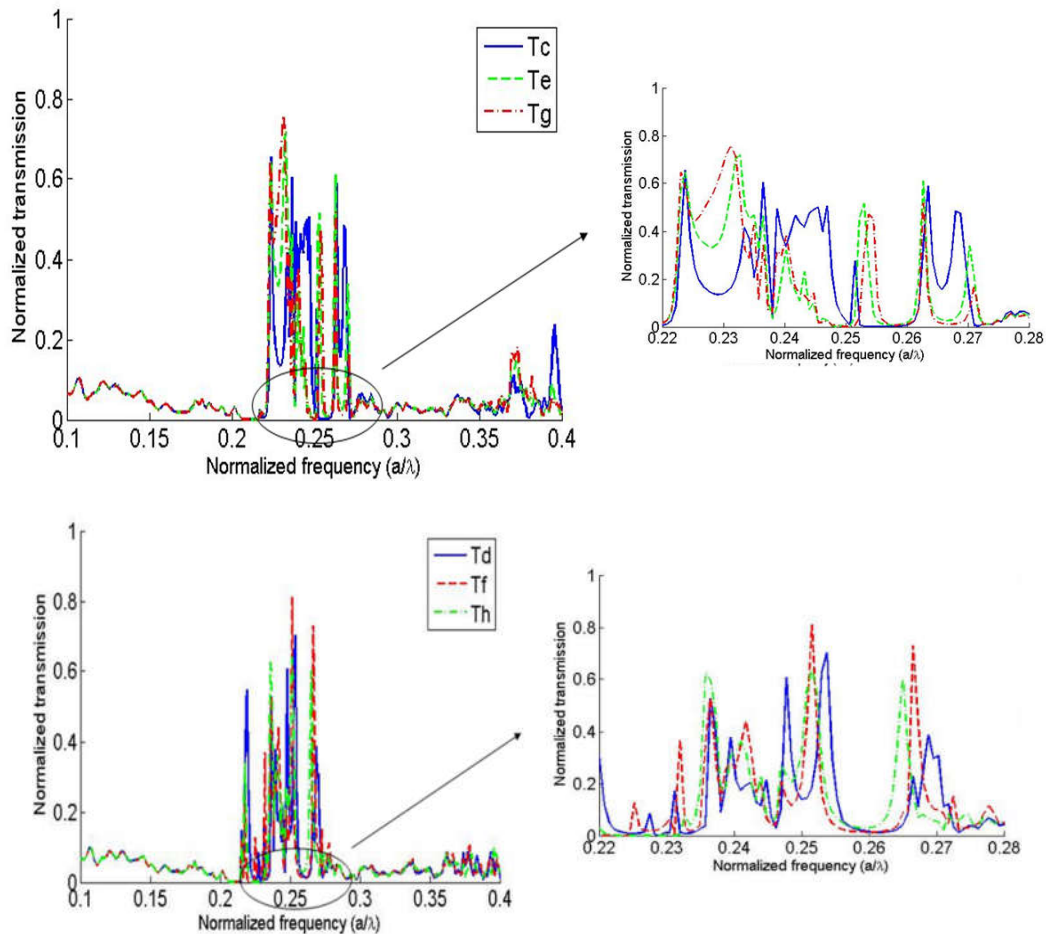


Figure 6 photonic crystals waveguide Z transition shown in Figure 5

As seen in Figure 6, structure b band gap has shifted than structure a, and the passage increased in  $0.24 \frac{a}{\lambda}$  and  $0.25 \frac{a}{\lambda}$  frequencies. The changes are resulting from the displacement of the cavities in the structure curvature. A comparison of the structures of c, e and g in Figure 6 shows that band gap structure c is wider than the e and g structures, The passage in the frequency range of  $0.23 \frac{a}{\lambda}$  for both e and g structures is increased than that of c, that the increase is higher for g structure. These changes are caused because we changed the radius of the cavity for these structures. For three structures of d, f and h, as can be seen in Figure 6, all three have wide band gap in frequency ranges of  $0.23 \frac{a}{\lambda}$ , to  $0.26 \frac{a}{\lambda}$ . Increase the rates of passage for these structures are, respectively: f, h and d, the changes as before are caused by changes in the radius of the cavity in the desired areas.

## CONCLUSIONS

In this paper we studied the photonic crystal waveguide Z by FDTD method. The waveguide efficacy and the wave frequency bandwidth through are important, here which we examine the passing curvature by placing different defects in the waveguide structure. According to the paper, defects caused broadening band gap and increasing structure pass at some frequencies, but location cavities leads to increased broadening of passing frequency band width as well as the amount of pass. Given the use of these structures, each of them can be used in place.

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