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Numerical Study Of Nonlinear Behavior Of A Sheared Immiscible Fluid Interface And The Possibility Of Drop Forming

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ABSTRACT
Kelvin-Helmholtz instability at the interface between two separate fluids that move in two opposite directions is considered. In the simplest condition, two similar inviscid fluids with a non-rotary flow are considered on both sides. In this article, Kelvin-Helmholtz instability has been studied for the interface between two non-mixing fluids and different density. Dimensionless parameters in this problem are Weber number and Reynolds number for studying the effect of surface tension and viscosity respectively. Numerical simulations were performed using a finite differential Front tracking method. The behavior of the interface is studied at different flow conditions. Specifically, the spreading of an applied disturbance wave on the interface is studied. The probability of drop formation is investigated in the flow. It is found that drop formation occurs at a specific range of Weber numbers. Ultimately, the effects of surface tension and viscosity on the development of the interface have been discussed.
KEYWORDS: Kelvin-Helmholtz Instability, Two-Phase Flow, Shear Flow, Surface Tension, Drop

INTRODUCTION

Flows with lower viscosities are more likely to become turbulent, while highly viscous flows would become turbulent in a longer period of time. In some cases, the instabilities of the flow increase and is amplified up to the point of bringing the flow into turbulent status. As an example when a layer of a relatively heavy fluid such as water moves on a lighter fluid layer such as oil, Rayleigh-Taylor instability would occur, leading the flow to become turbulent. Another instability called kelvin-Helmholtz occurs when two fluid layer move on each other with different velocities and in opposite directions, which would cause a turbulent flow as well. Kelvin-Helmholtz instability occurs when a shear stress is applied on a flowing fluid, or when two fluid layers flowing on each other have different velocities in the boundary layer between two flows. Examples of this phenomenon could be the displacement of the upper layer of water due to a blowing wind, the red point of the Mercury's surface, and the moving of the clouds and oceans' water (Fig. 1). This phenomenon was first described by two reputable physicists, Lord Kelvin and Hermann Von Helmholtz. The analysis and study of the linear stability in Kelvin-Helmholtz problem returns to the 19th century; however, researchers have recently performed numerous analyses of the nonlinear behavior of the concentrated vortices, in the field of fluid dynamics [1]. Given the development of computers and the possibility of rapid calculations, most of the researchers have conducted studies on very thin vortex planes consisting of two potential flow regimes. They have found out that due to the rapid growth of low mixtures in vortex planes, it is not quite easy to determine the exact formation point of the vortex.

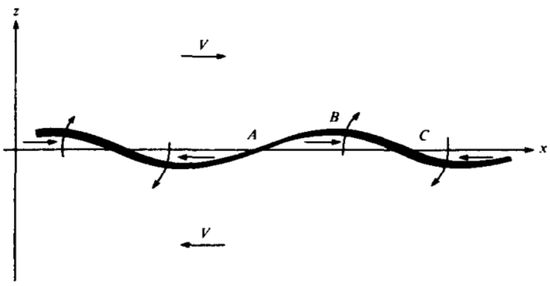

Figure 1. Kelvin Helmholtz instability. Two fluids movements in opposite directions causing turbulence in the flow

Krasny has shown that adjusting the vortex planes results in a better solution of the problem, in addition to the elimination of separate deformations [2]. Other researchers studied the nonlinear development of Navier-Stokes equations, and indicated that the turbulent shear layer finally results in thin vortices [3, 4]. Tryggyason and Dahm analyzed a specific range of large Reynolds numbers with small initial thicknesses, and compared the complete simulation of Navier-Stokes equations with adjustment inviscid planes of Krasny [5]. Kelvin-Helmholtz instability for two completely similar fluids with the possibility of mixing is currently considered as a solved problem [6]. In similar problems, the solubility of fluids with limited and separate density and viscosity has not been adequately studied, considering surface tension and non-mixing fluids. The effects of surface tension for fluids of equal densities, and by completely eliminating the effects of viscosity have been analyzed by Ho, Lungroob, and Shelly. They found that a high surface tension impedes the flow, and for this very reason, the interface of two fluids penetrate and grow in each other like fingers.

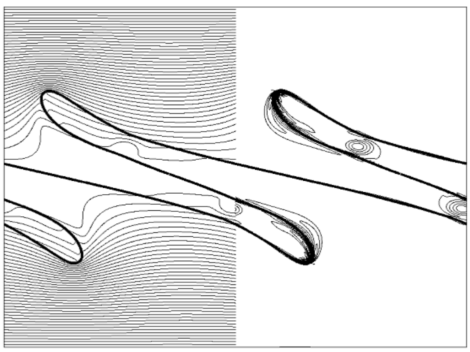

Figure 2. Deformation of the interface between two liquids due to surface tension and create finger

With low surface tension, the finger conditions of the interface would be more equal, and the interface of two fluids would behave similar to nonlinear classic Kelvin-Helmholtz instability [7]. Pozrikidis has done research about other ranges of Stocks flows, stating that the interface of two fluids would be less stable with increasing of viscosity rate, and would develop the elongated finger condition of the interface of two fluids [8]. Zaleski and Zenetti analyzed the effects of both inertia and viscosity including surface tension, leading to developments in finger condition interface for limited Reynolds numbers. Moreover, they indicated that the obtained results from finger condition in two dimensions can be applied for studying their 3D effects [9]. Currently, perceiving the segmentation and separation of the interface between two insoluble fluids with different properties is in the core of attention. In this paper, the effects of both surface tension and viscosity of fluids with different densities have been considered. The behavior of the interface between two fluids of different densities has been studied under various conditions.

GEOMETRY OF THE PROBLEM

The geometry of the problem is demonstrated in Fig. 3.

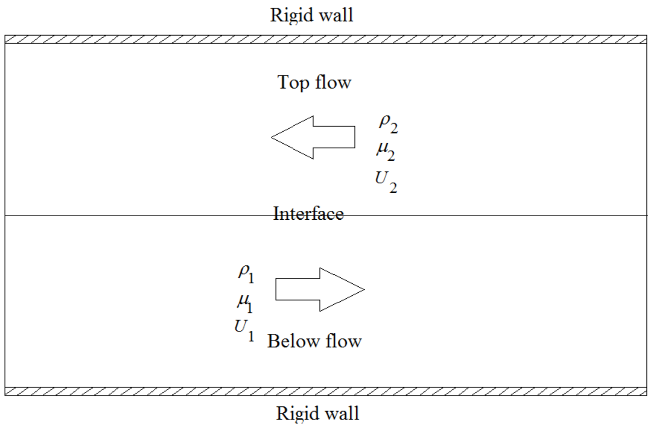

Figure 3. Schematic view of the motion of two fluids with different properties in different directions

Two different fluid layers, flowing in opposite directions, are in surface contact. The lower flow is a fluid with density ρ_1 and viscosity μ_1, while it is moving to the right with velocity U_1. The upper flow is a fluid with density and viscosity ρ_2 and μ_2 respectively, which is moving to the left with velocity U_2. The selected main axes are in the direction of the flow, and perpendicular to the direction of the flow. Moreover, the boundary conditions of the problem, in the direction of the flow, are periodic.

3. PROBLEM FORMULATION AND COMPUTATIONAL DOMAIN

In order to analyze the problem and develop a desirable computer code, a rectangular domain with periodic horizontal conditions and rigid moving walls in its upper and lower parts are considered. Dimensions of this domain are equal to 4λ in horizontal direction, and 2λ in vertical direction.

The interface development between two fluids can be determined by the velocity difference, surface tension, density, and the viscosity of fluids. When the viscosities are adequately small, it is expected that the initial growth rate be predictable by the linear stability analysis of inviscid fluids [10]. A disturbance is applied to the interface in the front which is a function of time t and position x:

$$A=A_0e^{st+ikx} \quad (1)$$

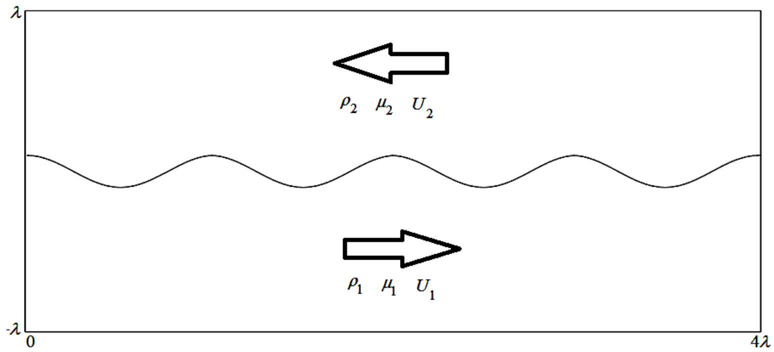

Figure 4. The computational domain

Considering the linear stability theory for two adjacent flowing fluids, with including the effects of surface tension, and neglecting the effect of gravity, the following equation would be obtained for S:

$$S = -ik\frac{\rho_1 U_1 + \rho_2 U_2}{\rho_1 + \rho_2} \pm \sqrt{\frac{k^2 \rho_1 \rho_2 (U_1 - U_2)^2}{(\rho_1 + \rho_2)^2} - \frac{Tk^3}{\rho_1 + \rho_2}} \quad (2)$$

where the first term is the phase velocity C, and the second term is the growth rate, defined as below:

$$S = \sigma \pm iC, C = \frac{-Im(S)}{k}, \sigma = Re(S) \quad (3)$$

Moreover, $K=\frac{2\pi}{\lambda}$ is the wave number, where λ is the disturbance wave length, applied to the interface of two fluids. In non-dimensional form, the growth rate and phase velocity would be obtained from:

$$\tilde{\sigma} = \frac{\sigma T}{\rho_2 \Delta U^3} = \frac{1}{We}\sqrt{\frac{1}{r+1}\left(\frac{r}{r+1} - \frac{1}{We}\right)} \quad (4)$$

$$\tilde{C} = \frac{C}{\Delta U} = \frac{1}{\Delta U}\frac{rU_1 + U_2}{(r+1)} \quad (5)$$

Here, the time scale and characteristic velocity are defined as:

$$\tilde{\tau} = \frac{1}{K\tilde{U}}, \tilde{U} = \Delta U \quad (6)$$

Where $\Delta U = U_2 - U_1$ and $r = \frac{\rho_1}{\rho_2}$. Weber number is defined as:

$$We = \frac{\rho_2 \Delta U^2}{TK} \quad (7)$$

where T is the surface tension between two fluids. The phase velocity clearly indicates that the initial wave is transferred via the average velocity of density concentration. The growth rate would have a real value, if the expression under the square root in Eq.4 retains a positive value. In other words, the minimum value of growth rate would be obtained by the following Weber number:

$$We > 1 + \frac{1}{r} \quad (8)$$

In addition the maximum value of growth rate would be obtained by the following Weber number

$$We_{max} = \frac{3}{2}(1 + \frac{1}{r}) \quad (9)$$

Moreover, Reynolds numbers would be defined, based on the properties of the upper and lower fluid layers:

$$Re_1 = \frac{\rho_1 \Delta U \lambda}{\mu_1}, Re_2 = \frac{\rho_2 \Delta U \lambda}{\mu_2} \quad (10)$$

The numerical method used for the computations presented here is based on writing one set of equations for the entire computational domain. This is possible by allowing for different material properties in the formulation and adding singular terms at the boundaries between the different fluids to ensure that the correct interface conditions are satisfied. The resulting "one-field" Navier–Stokes equations are:

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \rho \mathbf{u}\mathbf{u} = -\nabla p + \nabla.\mu(\nabla \mathbf{u} + \nabla \mathbf{u}^T) + T\int_F \kappa n \delta(x - x_f)da \quad (11)$$

Here, u is the velocity vector, p is the pressure, ρ and μ are discontinuous density and viscosity fields respectively, and n is the surface normal. The surface forces act only on the interface between the fluids and appears in the current formulation multiplied by a two-dimensional delta function, δ. The integral is over the entire front or interface. This equation contains no approximations beyond those in the Navier–Stokes equations and in particular, it implicitly contains the proper stress conditions for the fluid interface. Since the density and the viscosity are different for each fluid, it is necessary to track the evolution of these fields by equations of state, which specify that each fluid particle retains its original density and viscosity:

$$\frac{\partial \rho}{\partial t} = 0, \frac{\partial \mu}{\partial t} = 0 \quad (12)$$

The momentum equation is also supplemented by an equation of mass conservation, which for incompressible flows is:

$$\nabla.U = 0 \quad (13)$$

The momentum equation is discretized on a regular staggered grid using second-order central differences for the spatial derivatives and a second-order predictor–corrector time integration scheme. The continuity equation, when combined with the momentum equation results in a pressure equation that is not separable as for homogeneous flow and is solved by a multi-grid method [10]. To advect the material properties, and to evaluate the surface tension term in the momentum equation, we track the interface between different phases explicitly by connected marker points (front). The number of points representing the front is selected such that there are approximately 2–4 front points per stationary mesh. As the front deforms, points are added and deleted dynamically to maintain adequate resolution. The delta function is regularized by distributing the surface force and the density gradient onto the fixed grid. In the computations reported here, we have used a distribution function introduced by Peskin [11] which smooths the δ function to the nearest nine grid points. The one-field formulation used here is common to other techniques for multi fluid flows such as the VOF (volume of fluid) method and the more recent level set method. In these methods, however, the phase boundary is not tracked explicitly, but reconstructed from a marker function. Explicitly tracking the interface avoids the difficulty of advecting such marker functions and allows accurate evaluation of the surface tension. While very high surface tension can sometimes cause unphysical parasitic currents as well as stiffness problems, explicit tracking as well as a semi-implicit treatment of the surface tension helps minimizing these problems [12]. The method has been applied to a number of multi fluid problems and tested and validated in a number of ways, not only to check the implementation, but also its accuracy. Those tests include comparisons with analytical solutions for simple problems, other numerical computations, and experiments. The actual resolution requirement varies with the parameters of the problem. High Reynolds numbers, for example, generally require finer resolution than lower ones, as in other numerical calculations. However, in all cases we have found that the method converges rapidly under grid refinement, and in those cases where other solutions exist we have found excellent agreement, even for modest resolutions. For a more detailed description of the method, see Ref. 12.

LINEAR STABILITY

In the beginning, the velocity field is not continuous, and a large Reynolds number can be noticed; hence, the initial growth is expected to be completely predictable by inviscid analysis (Eq.2). In order to find the initial velocity field, the strength of vortex plane can be calculated by subtracting the disturbance velocities at the interface of two fluids. In non-dimensional form, the result would be as below [13]:

$$\tilde{\gamma} = \gamma \frac{kT}{\rho_2 \Delta U^3} = -\frac{1}{We} + \frac{r-1}{r+1}\frac{\tilde{\xi}\cos(\tilde{x})}{We} \pm 2\tilde{\xi}\tilde{\sigma}\sin(\tilde{x}) \tag{14}$$

Where $\tilde{\xi} = k\zeta$ is non-dimensional initial domain; $\tilde{\sigma}$ is linear growth rate in Eq.4, and $\tilde{x} = kx$ is non-dimensional horizontal position. Retaining the strength of vortex plane, the circulation Γ in separate points can be obtained as below, by integrating around a small area in the interface:

$$\Gamma = \int_{\Delta s} \gamma ds \tag{15}$$

In order to obtain the vorticity field, once the circulation is found for separate points of the interface between two fluids, it can be distributed to the static points of around the interface. Thereafter, the Stream function can be obtained by solving the following equation:

$$\nabla^2 \psi = -\omega \tag{16}$$

Finally, having the stream function, the velocity can be calculated from:

$$u = \frac{\partial \psi}{\partial y}, \upsilon = -\frac{\partial \psi}{\partial x} \tag{17}$$

After acquiring the initial velocity field, the solution of the problem would develop in time to get the evolution of the interface. In order to evaluate the non-dimensional linear growth rate, considering Eq.1 and Eq.4, by their combination, and rewriting them, the following equation would be obtained:

$$\tilde{\sigma} = \frac{T}{\rho_2 \Delta U^3}\frac{1}{t}\ln\left(\frac{A(t)}{A_0}\right) \tag{18}$$

Having the non-dimensional growth rate as a function of time, it would be possible to draw its diagram in each research step, and compare the results to those of linear inviscid theory.

5. RESULTS

In Fig.5, the non-dimensional initial growth rate is plotted as a function of time for two similar fluids (with equal densities), and a Weber number of 6 for a condition where Reynolds number for both of the flows is 10000. In this diagram, the predicted growth rate by the linear stability theory, and the growth rate obtained in this research have been demonstrated. The predicted growth rate by the linear stability theory can be obtained by Eq.4 as:

$$\tilde{\sigma} = \frac{\sigma T}{\rho_2 \Delta U^3} = \frac{1}{We}\sqrt{\frac{1}{r+1}\left(\frac{r}{r+1} - \frac{1}{We}\right)} = \frac{1}{6}\sqrt{\frac{1}{1+1}\left(\frac{1}{1+1} - \frac{1}{6}\right)} = 0.068 \tag{19}$$

If the growth rate remains constant, the diagram would be parallel to time axis as a full horizontal line, which indicates the initial growth rate obtained from linear inviscid theory. It is quite obvious that the numerical calculations in initial times converge to this line; however, as the wave amplitude increases, nonlinear effects reduce the growth rate.

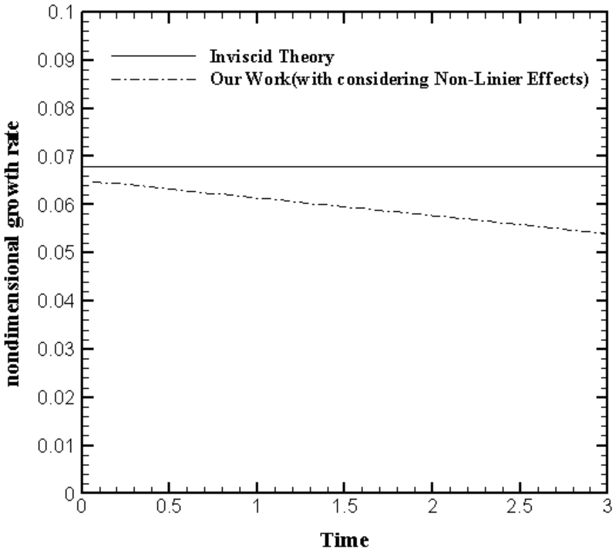

Figure 5. Initial non dimensional growth rate vs time for density ratio = 1, Weber number = 6 and

$Re_1 = Re_2 = 10000$

Repeating the calculations for numerous Reynolds numbers between 600 and 10000, it has shown that the changes are not highly considerable, which implies that the effects of viscosity are not significant in initial times. Moreover, this diagram indicates the validation of the present code for modelling this problem.
In order to study the effects of difference between two flows, we considered a density ratio 10. The non-dimensional growth rate from inviscid theory can be obtained:

$$\tilde{\sigma} = \frac{\sigma T}{\rho_2 \Delta U^3} = \frac{1}{We}\sqrt{\frac{1}{r+1}\left(\frac{r}{r+1} - \frac{1}{We}\right)} = \frac{1}{10}\sqrt{\frac{1}{10+1}\left(\frac{10}{10+1} - \frac{1}{6}\right)} = 0.027 \quad (20)$$

Figure 6 shows the growth rate versus time for this case.

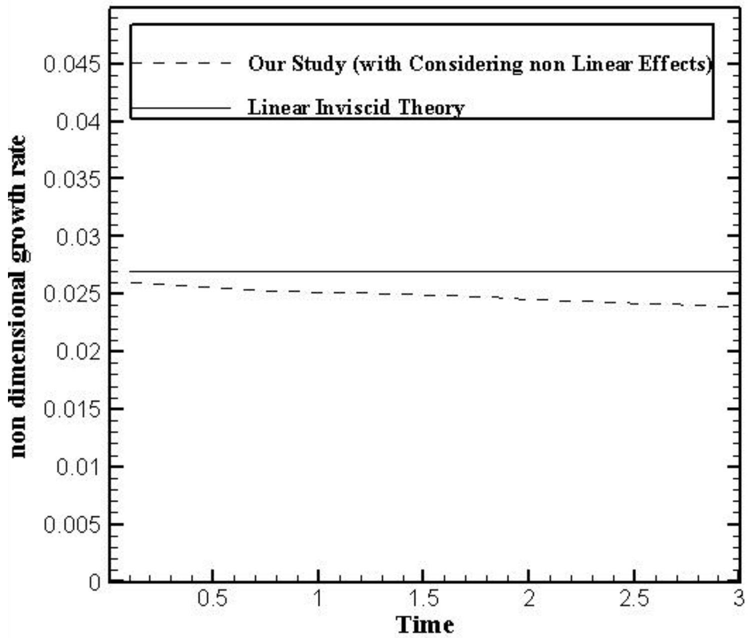

Figure 6. Initial non-dimensional growth rate vs time for density ratio = 10, Weber number=10,

$Re_1 = 5000$ and $Re_2 = 1000$

The development of the interface between two fluids with equal densities and viscosities has been demonstrated in Fig.7. In this calculation, the Weber number is 6 and the Reynolds number for both flows is 10000 and the grid resolution is 256×512. The initial wave amplitude (ζ) is 5persent of the wave length.

Figure 8 presents results of a simulation at a density ratio 10. Low Weber number is assumed (We=1.7). The Reynolds number of bottom and top flows are $Re_1 = 5000$ and $Re_2 = 1000$ respectively. The wave amplitude damps out after an initial growth.

In next step, we changed the Weber number to 5 (Fig.9). Here, drop formation is visible close to the end of simulation. Clearly a drop is likely to separate from the interface at the location where neck is formed.

Finally, we consider we=10 for studying the behavior of interface under very small surface tension (Fig.10). Here, the mechanism of fingering is more dominant, even though the evolution shows a narrow region of separation.

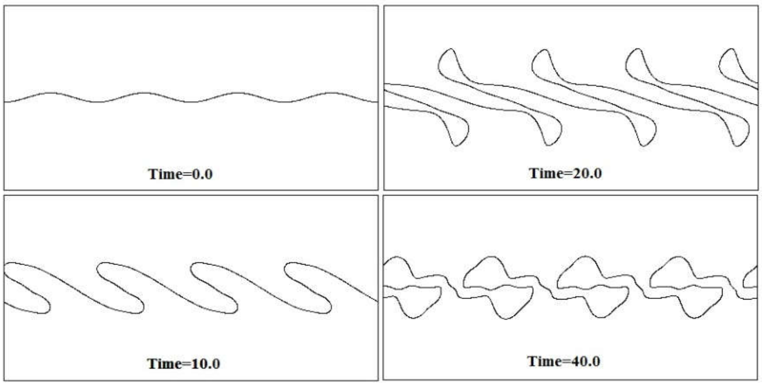

Figure 7. Kelvin Helmholtz instability and growth of disturbances acting on the interface of two similar flows for $Re_1 = Re_2 = 10000$ and We=6.0

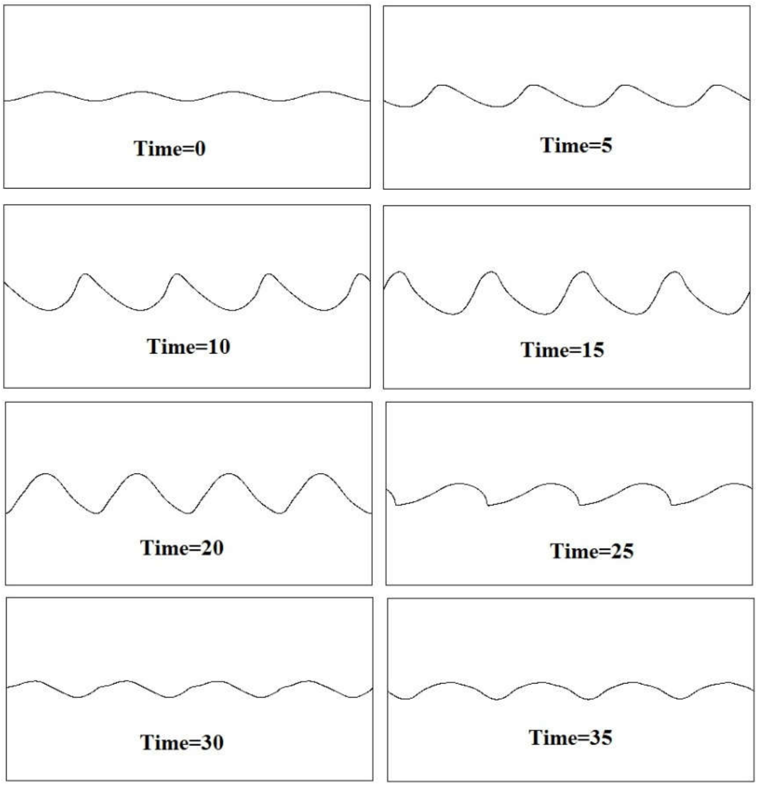

Figure 8. Kelvin Helmholtz instability and growth of disturbances acting on the interface of two different flows, density ratio = 10, $Re_1 = 5000$, $Re_2 = 1000$ and We=1.7

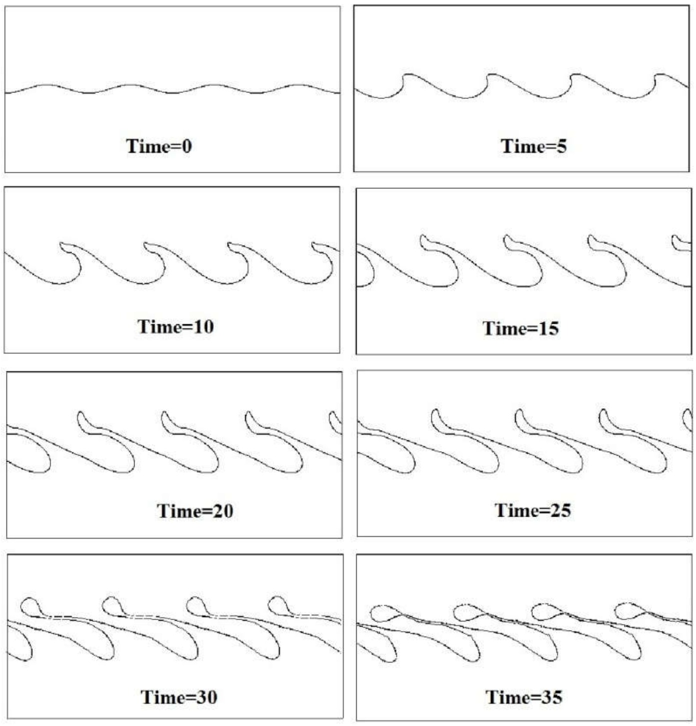

Figure 9. Kelvin Helmholtz instability and growth of disturbances acting on the interface of two different flows, density ratio = 10, $Re_1 = 5000$, $Re_2 = 1000$ and We=5.0

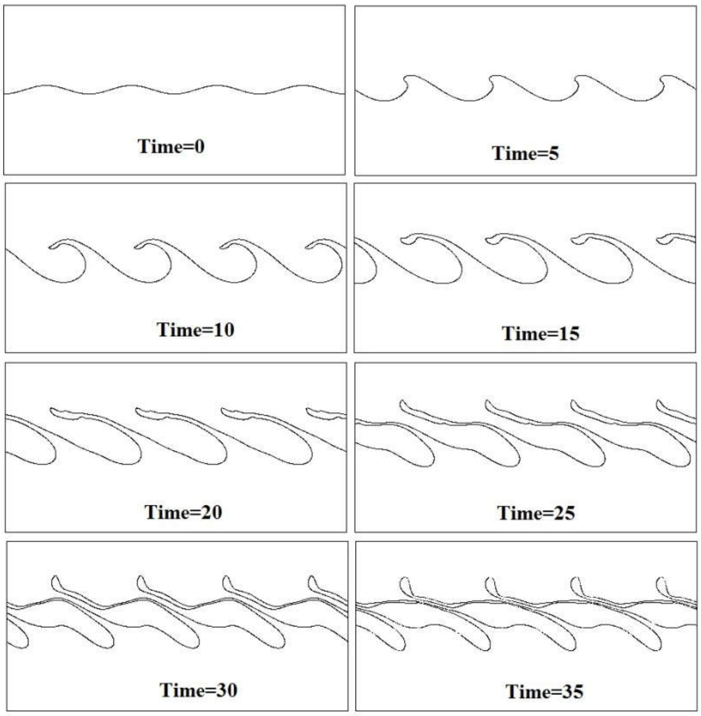

Figure 10. Kelvin Helmholtz instability and growth of disturbances acting on the interface of two different flows, density ratio = 10, $Re_1 = 5000$, $Re_2 = 1000$ and We=10.0

CONCLUSIONS

Several simulations of the two-dimensional Kelvin– Helmholtz instability of immiscible fluids are presented. The Reynolds numbers selected are sufficiently high so that the initial instability is well predicted by inviscid linear stability theory. It can be easily noticed that when we consider large Weber numbers, the initial disturbance grows rapidly. It has been observed that as the Weber number increases, the wave grows more rapidly, and the fluids grow into each other in a symmetric way for unity density ratio due to low surface tension.

For density ratio more than one, we can see asymmetric penetration in two flows. If the lower flow retains a higher Reynolds number, due to the reduction of its viscosity, it is noticed that the lower fluid penetrates into the upper fluid, and there is an asymmetry in the flow.
In this research,
It was attempted to find the condition where the interface would create discrete zones (drops), as it can be noticed at time 35 in Fig.9.
In all conditions that was explored here, this phenomenon is not observed for Weber numbers lower than 4. In other words, drop formation does not occur for Weber numbers below 3.
For Weber numbers around 6 (5 to 7), the interface is capable to form drops, and release it into the other fluid.
At larger Weber numbers (larger than 7), the growth rate of the interface is like fingers. these fingers or filaments form in the heavier fluid and release in the lighter fluid..

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