



Design Of Photonic Crystals Cavity For Application In Sensors

Tahmineh Jalali, Saeedeh Dehghan Nejad

Physics Department, Persian Gulf University, Bushehr 75169, Iran.

Email: jalali@pgu.ac.ir

ABSTRACT

In this paper, simulation of photonic crystal cavity for application in sensors will be studied by using finite element method (FEM). These sensors are used as efficient tools for measuring quantities such as temperature, pressure and humidity particularly in absorption of biological materials such as proteins, cells, viruses, particles and bacteria. In this paper, by considering the improvement of cavity quality factor, it's possible to significantly increase resonance modes area interaction with attached small molecules, we examined the transmission of electrical energy versus frequency. Finally, the quality factor of these structures was calculated.

Keywords: photonic crystal, cavity, quality factor, sensor, finite element

INTRODUCTION

Over the past two decade, significant activities for the development of photonic devices that can limit and control light, have been carried out. The key motive for doing this is to understand photonics area which recently has become close to modern electronics. Therefore, many researchers have been performed on photonic crystals as a proper candidate for the construction of optoelectronic elements [1].

These crystals are periodic arrangement of materials with different refractive indices which can be along one, two and three directions [2]. Their unique property is the existence of band gap, which is a certain frequencies by which, light, is not allowed to emit [3].

By creating defects in photonic crystals, tools such as waveguide, cavity or transducer can be made. Waveguides are linear defects and cavities are different types of point defects in photonic crystals. Each of these point defects, bring about different applications of cavities in tools such as lasers, light emitting diodes (LED) and particularly sensors. These structures are suitable in lasers because they give high performance resulting from localizing light in a small area, due to localizing light, it's possible to increase spontaneous and inductive emissions, therefore, it's highly effective on the laser output power and lasing threshold. In LEDs, due to the importance of spontaneous emission, by creating the high quality factor in cavity, it's possible to increase this quantity and decrease loss. With the increase of quality factor, diode brightness increases. Also in sensors, with the increase of defect area, it's possible to cause strong interaction between cavity resonance mode and the material existing in defect holes, and therefore increase the sensitivity of sensors. Moreover, due to the properties of photonic crystals such as emission or reflection, these cavities are also effective in biological sensors, and this paper investigates these very cavities.

Currently, for sensors in microchips scale, electrochemical systems with good sensitivity or even more sensitivity than common tools have been developed. Also with the development of optical waveguides, micro-cavities and other micro-structures, optical sensors in microchip scale emerge in various areas specially biology, mechanics and chemistry [4].

Given the high cost of the construction of photonic crystal tools, accurate designing and modeling with the use of computational techniques, has become more importance. In this regard, in this paper, first, photonic crystal cavities are examined and then with the selection of a proper numerical method, problems related to these structures are solved. It should be noted that these methods are in the two areas of time and frequency. In the area of time, methods such as finite difference and finite element and in the area of frequency, bed expansion function, veneer functions and multiple multipole method (MMM) and finite difference and element (of time area), exist. Among these methods, FEM is a suitable method to solve electromagnetic problems (even with complex geometries and nonhomogeneous environments) based on

frequency, and this method has been chosen as the basis for this paper. Moreover, this method is useful in antenna, microwave circuit, motor and generator modeling. In the following, different simulations of the selected cavities quality factor (with multiple holes defect) are explained and in the end, the obtained results will be discussed.

2- Cavity

Much effort has been dedicated to understand different methods through which photonic crystals can reflect and trap light. For such applications, the existence of defects in them that break the period of dielectric function and localize light, is necessary [5].

Cavities are created by causing defects in photonic crystals, in a way that they're constructed with the removal of one or more column shafts, of course, this structure must have a proper size to trap light (figure (1)). In such structures, light is limited by the composition of Bragg reflection and the internal reflection inside the defect area [6].

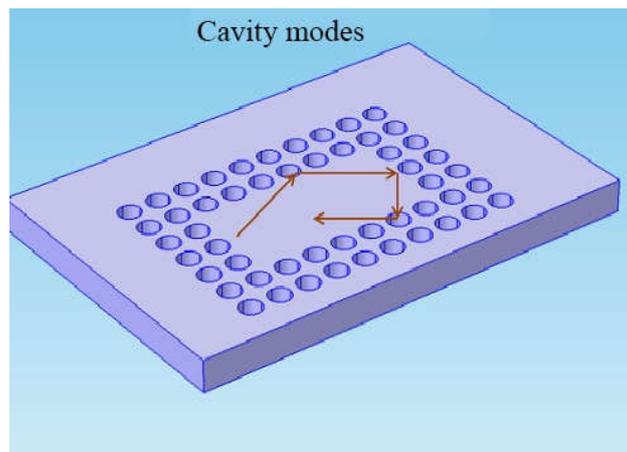


Figure (1): a trapped defect mode in a cavity sample

In figure (1), it's possible to create a cavity surrounded by reflective walls by removing several columns in photonic crystals. By trapping electromagnetic wave in unipolar, bipolar, or four-polar different modes defect area, they appear.

The simplest method to build cavity, is to change the radius or the refractive index of holes in structure. By adjusting the radius of the shafts, the frequency trapped in cavity can be adjusted. This adjustment capability, is one of the important properties of photonic crystals. In fact, resonance cavity is only useful when we want to control reflection in a frequency area. Three important properties of cavity are frequency, half-life and symmetry. Frequency can be controlled by changing cavity geometry. Also, Q can be controlled by changing the period of crystal, band gap size or cavity mode frequency. Photonic crystal cavities, are recognized based on the two parameters of quality and resonance mode volume. Quality factor (Q), is a scale that shows how many oscillations should occur in a cavity before the wave is weakened, this quantity is highly dependent on the depth of the holes constructing the crystal and the refractive index of its surrounding material. These cavities are suitable for the controlling of reflection in a narrow frequency area. Also, by changing the geometrical parameters, they can be optimized for use in tools. Today, different cavity applications such as interactions of coherent electron-hole, extremely small filters, low threshold lasers, high sensitivity sensors, photonic chips, nonlinear optics and switches can be named.

3- Sensor

Over the past three decades, optical sensor elements have been vastly researched and up until today, new technologies have been introduced. Currently, the application of photonic crystal sensors has been highly efficient for biosensing. These applications include identification of DNA, protein, cells and bacteria. Regarding this issue, some commercial sensors have been created [7].

Sensors work as a tool for turning especial chemical or physical phenomenon into electrical signal. A sensor changes physical quantities such as pressure, heat, humidity, temperature and etc. into electrical quantities. Such signals should enter through a sensor, and when the sensor changed the information being measured into electronic signal, the signals will enter the sensor, this part will filter and strengthen the electronic signal to produce an electrical signal that can run output devices.

Different properties of photonic crystals, makes them appropriate for use in optical biological sensors. These crystal structures can be designed for electromagnetic field emission with high localization, also the ability of these crystals in coupling light amplification with high quality, electromagnetic energy density and strong optical limitation, can be really useful for the production of high sensitivity biochemical sensors.

The cavity introduced in this paper, is examine with the use of multiple-hole defect for use in biosensors. Photonic crystal biosensors that have defect cavities, improve Q and sensor sensitivity.

4- Numerical finite element method

Up until about 1940 A.D, electromagnetic problems used to be solved by analytical methods, but with the coming of computers, numerical methods gradually became important and replaced classic methods. There are different numerical methods to solve differential equations in scattering problems that are used based upon the type of the problem and the ability of the numerical methods. The numerical method used in this paper, is the finite element method (FEM). This method, is a method to solve boundary value problems that is used in various problems such as mechanical engineering and electromagnetic waves [8]. The systematic principles of the method, provide the possibly of building general-purpose computer programs for solving a wide range of problems [9]. Because digital computers deal with discrete numbers, indirect solving of integral and differential problems is impossible. Therefore, we should divide problem space into several volume elements. The basis of this method is to replace the integrated field, with a number of subfields that that are shown with interpolation function with unknown coefficients. Therefore, the problem of infinite freedom boundary value changes into a problem with finite number of freedom degree. In other words, system solution is estimated with a limited number of unknown coefficients. An algebraic equation system is obtained and in the end, the boundary value problem will be solved by the equation system solution. Therefore, the finite element investigation of the boundary value should consist of four stages. To conduct these stages, first the problem area will be divided or separated into a number of finite elements or subfields, like in figure (2). In the next stage, with the selection of interpolation functions and summarization of all the elements in the problem area, the obtained equation system will be solved.

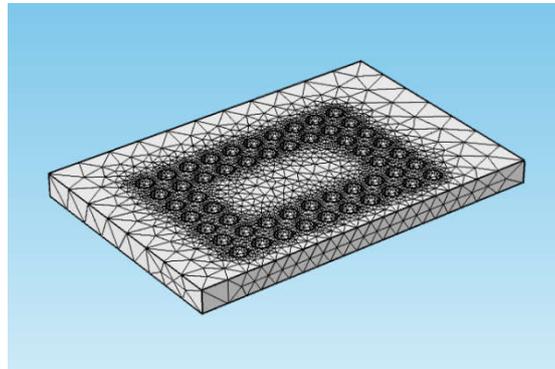


Figure (2) a sample of finite element separation

Area separation, is the first and most important stage in the finite element analysis, because the area division method affects the required computer memory, computation time and numerical results accuracy. The equation, governing the analysis of photonic crystals and obtaining of its modes, are the Maxwell equations ((1-4) to (4-4)).

$$(1-4) \quad \nabla \times E(r, t) + \frac{1}{c} \frac{\partial H(r, t)}{\partial t} = 0$$

$$(2-4) \quad \nabla \cdot D = 4 \pi \rho$$

$$(3-4) \quad \nabla \times H(r, t) - \frac{1}{c} \frac{\partial D}{\partial t} = \frac{4\pi}{c} J$$

$$(4-4) \quad \nabla \cdot B = 0$$

In these equations, $D = \epsilon E + P$, $B = \mu_0 M + \mu_0 H$. Here E and H are electric field and magnetic field, respectively, and D is electric displacement, ϵ is electric permittivity constant, μ_0 permeability constant, P polarization density, J free current density and ρ is free charge density.

Configuration is described by the ϵ dielectric function that is a function of location in space. Here, linear dielectric materials ($P=0$), are independent from ϵ , and are nonmagnetic ($M=0$). ϵ value, is real due to ignoring light absorption by materials. Also ρ and J are zero, and are assumed as one for most magnetic permeability materials. By considering the above issues, the equations (5-4) and (6-4) will be obtained.

$$(5-4) \quad D(r) = \epsilon(r)E(r)$$

$$(6-4) \quad B = H$$

In the end, by making some changes in the equations ((1-4) to (6-4)), the main equation for photonic crystals will be obtained ((7-4) equation).

$$(7-4) \quad \nabla \times \left(\frac{1}{\epsilon} \nabla \times H(r, t) \right) = \frac{\partial^2}{\partial (ct)^2} H(r, t)$$

The mentioned equations can be solved given the boundary conditions and Maxwell equations governing photonic crystals of electromagnetic problems, and examples such as some boundary conditions including magnetic field, perfect magnetic conductor, adaption, period and dispersion can be mentioned.

The condition of magnetic field, identifies the tangential component of the magnetic field and is usable in internal and external boundaries. The magnetic conductor boundary condition that is used for the upper and lower boundaries of cavity configuration in this article (green arrows in figure (4)) is defined as the tangential component of the zero magnetic field.

$$(8-4) \quad H \times n = 0$$

Flow density in inner boundaries cannot be determined when we solve the problem for the magnetic field. This condition is a special case of surface flow boundary condition. The adaption condition is for boundaries that show no boundary condition, also this condition, makes the boundary, completely unreflective. Periodic boundary condition, is naturally proportional to zero flux. There are three boundary conditions that are integrated periodic, anti-period and Flow Quant. The dispersion boundary condition is used for the left and right sides of the cavity configuration simulation in this paper (red arrows in figure (4)). This boundary condition, is transparent for the bed wave reflected on it. In fact whenever we want the boundary for the dispersed wave be transparent, we use this condition. Equation (9-4) shows a wave sample usable for this condition.

$$(9-4) \quad E = E_{sc} e^{-ik(n.r)} + E_0 e^{-ik(K.r)}$$

For the boundary condition to be completely transparent, it boundary should be an open boundary. Also, this condition holds true for spherical and cylindrical waves. It should be noted that the left side of the above equation, is the dispersed bed wave (equation (10-4)).

$$(10-4) \quad E = E_{sc} e^{-ik(n.r)}$$

5- Quality factor

Since comparison of cavities from their apparent form is difficult, cavity quality factor is introduced. In physics and engineering, a parameter quality factor is dimensionless that explains how an oscillator or resonator becomes damped. In other words, the width of the resonator band, is identified by its own central frequency. When a resonance cavity is designed for a specific application, usually radiation losses are minimized or similarly radiation quality factor is maximized. The mode existing in the resonance cavity, slowly dampens, and acts as the mode with the mixed frequency $\omega_c = \omega_0 - i\gamma/2$, which is the imaginary part of the frequency proportional to an exponential dampness. If field dampness is $e^{-\gamma t/2}$, then

the cavity energy will be dampened as $e^{-\gamma t}$. Level of dampness can be identified by γ , but since they're invariant in Maxwell equations, we prefer to show it with the dimensionless value of $Q = \omega_0 / \gamma$. This value, is known as the quality factor which can be interpreted in different ways. The method used in this article, is the calculation of the $1/Q$ proportion. Resonance cavities have discrete oscillatory frequencies with a fixed field for each resonance frequency. Therefore, if the aim is to excite a specific oscillatory mode in the cavity, then the fields only connect with each other when the excited frequency, is exactly equal to the resonance frequency of the selected mode. The resonance diagram has ω_0 in the maximum half, which equals ω_0 / Q (figure (3)).

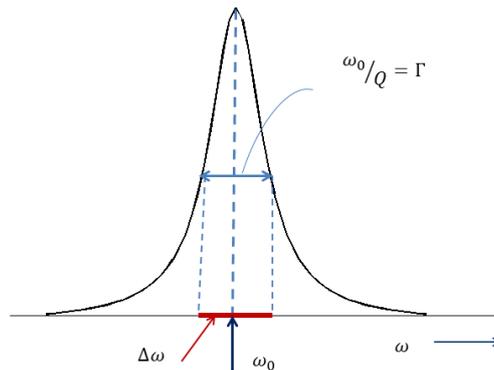


Figure (3): Illustration of the resonance peak and its relation with quality factor

Figure (3) for the constant input wave, the energy oscillations curve in the cavity as a frequency function, near the specific frequency, shows the resonance. Therefore, the separation frequency of $\delta\omega$, among points with half the power, identifies the Γ width, and cavity Q will equal:

$$(1-5) \quad Q = \frac{\omega_0}{\delta\omega} = \frac{\omega_0}{\Gamma}$$

Quality factor is reversely related to the peak width. The highness of the defect modes quality factor, is resulted from the fact that the domain of its wave function, with a distance from the defect structure center, dampens. Therefore, strong coupling between defect modes and the radiation field of the crystal output, can become infinitively small. Resonance mode quality factor, is a function of the number of holes on the cavity. The width of the defect mode, quickly reduces with the increase of the layers around the defect. Low quality factor cavity can perform on a wider frequency area, and it may be more stable, and show less sensitivity towards input disturbances.

6- Simulation configuration

The aim of this paper is to simulate a two dimension photonic crystal cavity, based upon which, the total electrical energy level passing from the structures are investigated with the finite element method and selected boundary conditions. For this purpose, the photonic structure with a hexagon network of air holes in the field environment with the infraction index of 3.4 is simulated. The configuration is shown in figure (2). The selected defect, is replaced by removing one of the holes and replacing it with a number of micro-holes. The constant of the (a) network is set to one, radius of the holes is set to the triple amount of a. The effective radius of defect holes are selected are 0.2a (figure (5)), 0.3a (figure (6)), and 0.4a (figure (7)), respectively. The effective radius is the area that involves the defect, and in each of the three models, the radius of the defect micro-holes are 0.04a and with the distance of 0.12a from each other. Also the range is selected as 5×10^7 (Hz) to 8×10^7 .

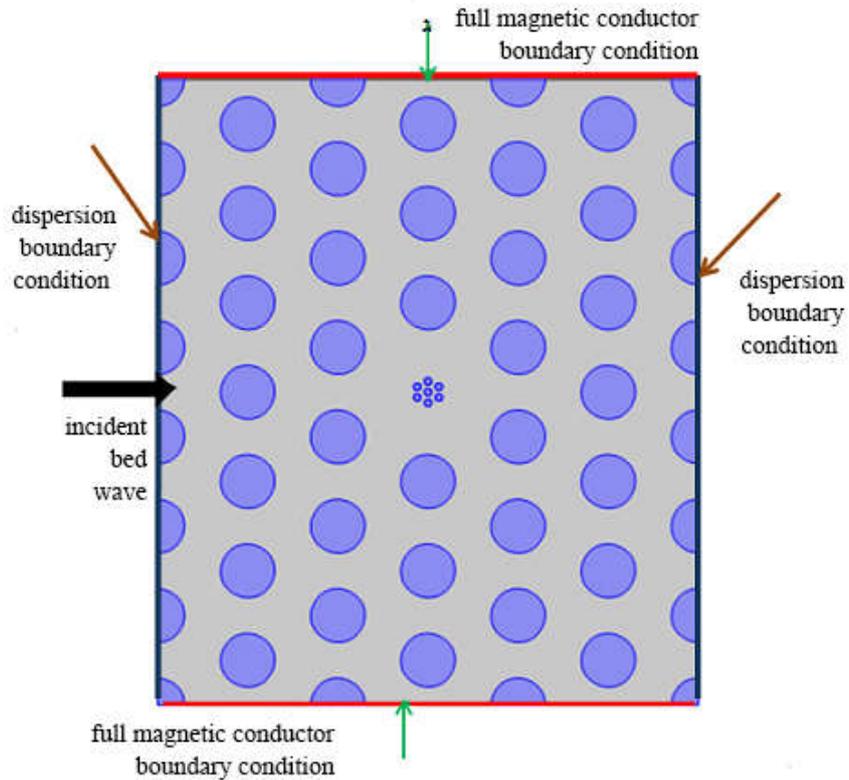


Figure (4): Cavity simulation configuration

Figure (4) shows cavity simulation configuration, in which the black arrow shows the selected boundary for the passing of electromagnetic wave with a domain of $1A/m$, the upper and lower green arrows, show the full magnetic boundary condition and the red arrows show the dispersion boundary condition. The configuration of two other cavities are the same, and they only differ in the number of micro-holes.

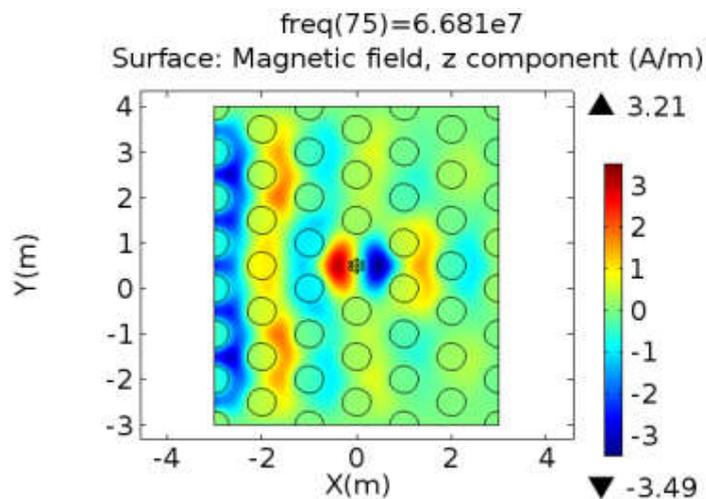


Figure (5): the illustration of the electromagnetic field in crystal for the frequency of 6.684×10^7 Hz. Photonic crystal, with multiple-hole defect with an effective radius of $0.2a$, refractive index of the field material of 3.4 , refractive index of the holes of 1 , holes' radius of $0.3a$, and defect holes' radius of $0.04a$, and they're placed next to each other with a distance of $0.12a$. In figure (5), the hole is replaced with 7 micro-holes, and the illustration of the resonance for this mode, occurs in the 6.681×10^7 . In the defect area, the electromagnetic wave is trapped.

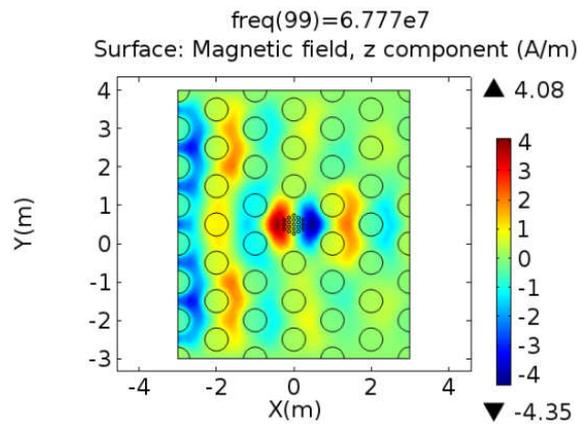


Figure (6): the illustration of the electromagnetic field in crystal for the frequency of 6.777×10^7 Hz. Photonic crystal, with multiple-hole defect with an effective radius of $0.3a$, refractive index of the field material of 3.4 , refractive index of the holes of 1 , holes' radius of $0.3a$, and defect holes' radius of $0.04a$, and they're placed next to each other with a distance of $0.12a$.

In figure (6), the hole is replaced with 19 micro-holes, and given the figure above, and the illustration of the resonance mode, the resonance occurs in the 6.777×10^7 Hz.

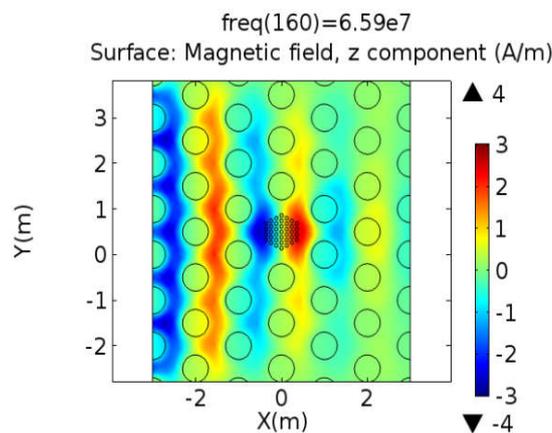


Figure (6): the illustration of the electromagnetic field in crystal for the frequency of 6.59×10^7 Hz. Photonic crystal, with multiple-hole defect with an effective radius of $0.4a$, refractive index of the field material of 3.4 , refractive index of the holes of 1 , holes' radius of $0.3a$, and defect holes' radius of $0.04a$, and they're placed next to each other with a distance of $0.12a$.

In figure (7), the illustration of resonance mode for this mode, has been shown with 36 micro-holes, and the resonance occurs in the frequency of 6.59×10^7 . Photonic crystal cavities are used in sensor devices which are highly used today. Since these structures have a unique feature in the controlling of light radiation, they leave a remarkable effect on the sensitivity of the sensors. Also, due to the high quality factor, and strong interaction of the electromagnetic fields coupled with material surface, sensor sensitivity can be increased in these cavities. Finally, in figure (8), the diagram for total electrical energy based on frequency is drawn. By calculating the width of the defect mode in the maximum half and by using equation (1-5), the cavities quality factors can be obtained. For the cavity with an effective radius of $0.2a$, this quantity is calculated as being about 105 and for the cavity with an effective radius of $0.34a$, it's about 76 and for the cavity with an effective radius of $0.04a$, it's calculated as about 73. Also in figure (9), this same simulation has been illustrated based on defect holes with different refractive indices.

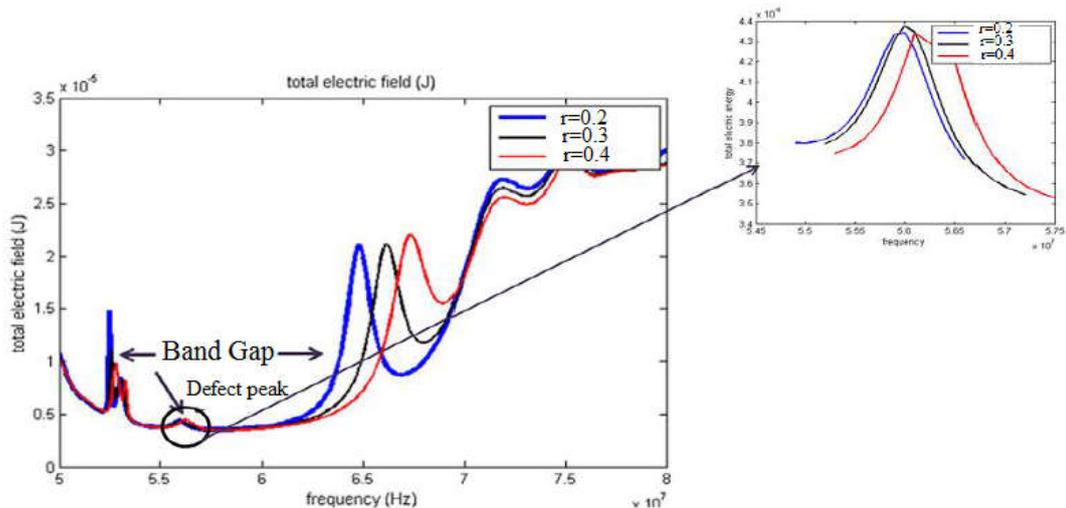


Figure (8): Diagram of total electrical energy based on frequency

Figure (8) is the diagram of total electrical energy per frequency based on 5×10^7 to 8×10^7 (Hz) range, for photonic crystal structures with multiple-hole defects and effective radii of $0.2a$ blue curve and $0.3a$ black curve, and $0.4a$ red curve. The black curve shows the gap band in the structures. According to the diagram it can be said that the illustration of the modes are similar in these structures and the right hand side diagram, is the enlargement of defect mode areas and there's one displacement in the formation place of the resonance mode.

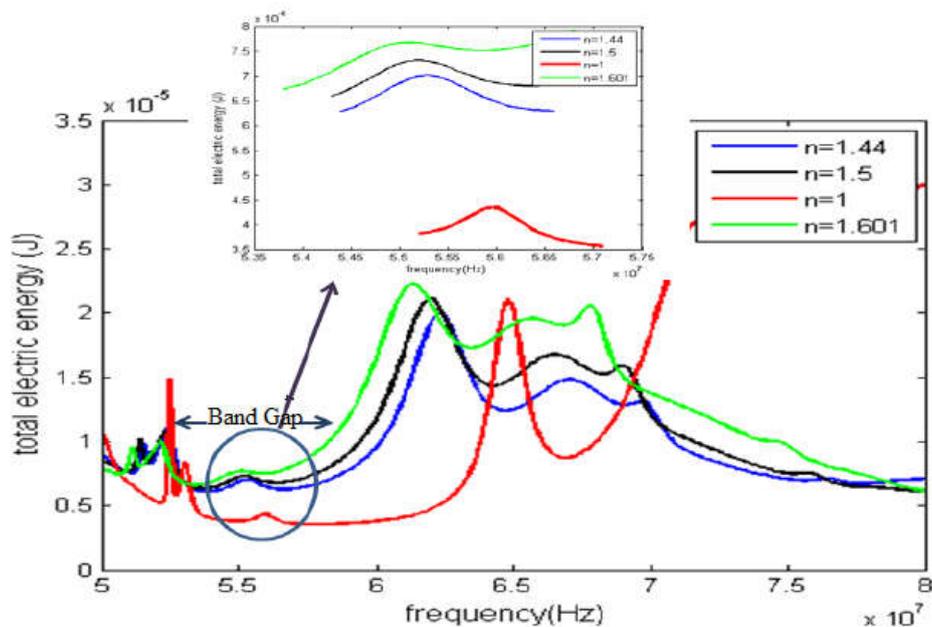


Figure (9): illustration of electrical energy based on frequency

In figure (9), the results of this same simulation for defect materials with different coefficients and the comparison of this configuration with the air (red curve) are shown. The selected materials of the two oil materials have refractive indices of 1.44 (blue curve) and refractive index of 1.601 (green curve) and benzene with a refractive index of 1.5 . According to the diagrams, these three materials (the two oil materials and benzene) show different behaviors compared to the air, because they have a bigger defect area and smaller gap band. Also, the upper diagram in figure (9), shows the enlargement of the identified area (circle sign), and with the increase of the holes refractive indices, we experience the displacement of electrical energy towards higher energies and widening of the defect mode area.

CONCLUSIONS

In this paper, a cavity for use in sensors is created by creating multiple-hole defect in photonic crystal. By drawing the image of the electric field for different frequencies in the selected structures, different resonance modes are created. In the next stage, we calculated the quality factor, and by comparing this quantity in our selected cavities, we experienced the increase of quality factor and decrease of effective radius. For photonic sensors, given the sensitivity of the structure towards the refractive index of the materials, by changing the effective radius of MHD, a structure can be simulated in which, quality factor and sensitivity, will be balanced.

Also by changing the effective radius of the defect hole and the distance between them, sensitivity and quality factor can be modified. These photonic crystals sensors, are really suitable for the identification of very small molecules, and it seems that they're a proper replacement for traditional sensors.

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