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# Fractional SEIR -SEI Epidemic Model on Zika Virus

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**ABSTRACT**
*Zika virus is one of the flavivirus epidemic which strikes,specially the pregnant women and toddlers, through Aedes mosquito. The aggregate affected human population is divided into susceptible, exposed, infectious, and recovered (SEIR).Therefore, a mathematical model named SEIR-SEI has been proposed to predict the number of infected individuals. The subject work illustrates the 2016 eruption of Zika virus in Brazil through the advancement of Conformable Fractional Epidemic model. This article proposes afractional epidemic SEIR-SEI model and exhibits a comparative study of the number of susceptible individuals with Conformable Fractional Epidemic model, through some graphs.*
***Keywords:*** *Epidemic model, SEIR-SEI Model, Zika virus, Conformable Fractional Derivative.*

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## INTRODUCTION
The Zika virus infects humans tocauseZika fever.The symptoms seen in the patient includes sudden onset of fever with the molecular or popular rash and arthritis [1, 2]. Zika virus is transmitted by the mosquitoes Aedes [3]. Report have shown that men who have recovered from Zika fever can still transmit the virus to its partner through his semen for up to the seven weeks after recovery from illness. The first documented case of dated back to 1947 in the Zika forests of Uganda, hence the name is Zika virus disease. and was later found in 1968 in Nigeria. About April 2015, first Zika virus authochthonic case was reported in Brazil [4], [5], [6]. Nearly 87 countries have reported mosquito born Zika transmission as of july 2016 [7]. Agencies investigating the Zika virus outbreak are finding an increased body of evidence about the link between Zika and microcephaly. National health authorities in Brazil has reported potential neurological and auto immune complication of Zika virus disease. There is also the scientific consensus about the same in the World Health Organization [9]. To control the spread of disease, require a dependable mathematical model which can accurately forecast the spread of disease during the outbreak and which can be put to use practically along with the modalities of disease control program[10, 11, 12] of various agencies.

This paper is organised in four sections. In Section 2, the mathematical model is described and in Section 3, the basics of fractional calculus have been introduced. Section 4 compares the graphs of the various frame works and finally in Section 5, the main contribution of this work is emphasized and a path for future work is suggested.

## MODEL EQUATIONS:
The following (non-linear) autonomous system of ordinary differential equations defines the evolvement of infected individuals through the SEIR-SEI group.

$$\begin{aligned}
\frac{dS_h}{dt} &= -\beta_h S_h I_v \\
\frac{dE_h}{dt} &= \beta_h S_h I_v - \alpha_h E_h \\
\frac{dI_h}{dt} &= \alpha_h E_h - \gamma I_h \\
\frac{dR_h}{dt} &= \gamma I_h \qquad (1) \\
\frac{dS_m}{dt} &= \delta - \frac{\beta_m S_m I_h}{N} - \delta S_m \\
\frac{dE_m}{dt} &= \frac{\beta_m S_m I_h}{N} - (\delta_m + \delta) E_m
\end{aligned}$$

$$\frac{I_m}{dt} = \alpha_m E_m - \delta I_m$$

$$\frac{dC}{dt} = \alpha_h E_h$$

While the total human population is depicted by N, $\frac{1}{\alpha}$ indicates the incubation period ("h" corresponding to humans and 'm' for mosquitoes). $1/\delta$indicates the life cycle of mosquito and $1/\Upsilon$ is tthe transmission rate, specifically when $\beta_h$ is the mosquito to human rate, $\beta_m$ is the human to mosquito rate.

On the right hand side the first δ of$\frac{dS_m}{dt}$ indicates that there is no change in the total mosquito count during the entire calculation. The$\frac{dC}{dt}$ equation evaluates the total number $c(t)$ of the affected people till the time t; that is the total number of individuals that has contracted the disease so far or has earlier infected or is in the infectious group in the given time of population that contacted the disease so far and has passed through or is in the infectious group at the given time. This is the Classical Integer model.The graph of susceptible human beings over the 50th week of outbreak using the parameters from Table 1 is given below as Figure 1:

.

**Table 1: Table of values of the parameters**

| Parameters | Values | References |
|---|---|---|
| $\propto_h$ | 1/5.9 | [13, 14, 15, 16, 17] |
| $\propto_m$ | 1/9.1 | [18, 19 20] |
| $\gamma$ | 1/7.9 | [17, 20 21] |
| $\beta_h$ | 1/11.3 | [ 20] |
| $\beta_v$ | 1/8.6 | [20 ] |
| $S_h$ | 205, 953, 959 | |
| $E_h$ | 8,201 | |
| $I_h$ | 8,201 | |
| $I_m$ | $2.2 \times 10^{-4}$ | |

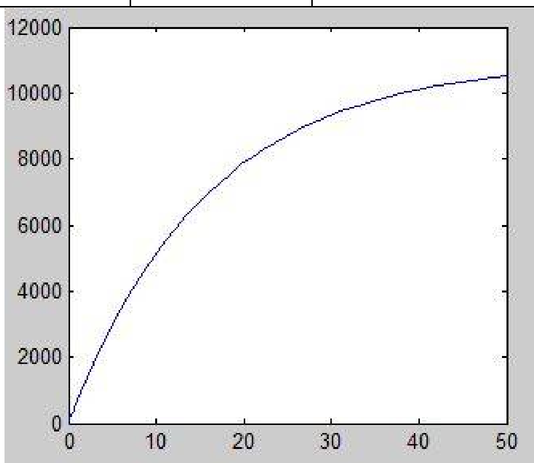


**Figure 1: Susceptible Humans over 50th Week as per Integer Order Model**

In Figure 1,$X$axis shows the time period in weeks, Y axis reflects the number of susceptible humans.

**CAPUTO -FRACTIONAL DERIVATIVE:**

Even though the Fractional Calculus is established mainly in the pure branch of Mathematics, since it can give a more realistic interpretation of the real problem, this has found application in engineering, bio and allied sciences in the recent era [22].The differential operators used in fractional calculus are non-integer or fractional order, which has memory implications and is useful for demonstrating many natural phenomena. To study of epidemiological, if we look at the development of dynamical growth It is appropriate to include memory effects because framework confined on stability memory that is limited by the order of the fractional derivative operator. At the present time many investigators have investigated and recommended well planned technique to examine the real and approximate solutions to the differential equation with a fractional operator. Many researchers are studying epidemic models which are related to distinct infectious disease contain fractional operator because it revel the reasonable physic deterioration of contamination of disease. The most popular definition of fractional derivative may be defined byCaputo as under:

$$ {}_{a}^{c}D_t^{\propto} = {}_{0}D_t^{-(n-\propto)} = \frac{1}{\Gamma(n-\propto)} \int_0^t (t-\tau)^{n-\alpha-1} f^n(\tau)d\tau. \quad (2) $$

Now we will consider the Caputo Derivative since it may be combined with the Classical Integer model.

$$ \begin{aligned} {}_{a}^{c}D_t^{\propto_1} S_h &= -\beta_h S_h I_v \\ {}_{a}^{c}D_t^{\propto_2} E_h &= \beta_h S_h I_v - \alpha_h E_h \\ {}_{a}^{c}D_t^{\propto_3} I_h &= \alpha_h E_h - \gamma I_h \\ {}_{a}^{c}D_t^{\propto_4} R_h &= \gamma I_h \\ {}_{a}^{c}D_t^{\propto_5} S_m &= \delta - \frac{\beta_m S_m I_h}{N} - \delta S_m \\ {}_{a}^{c}D_t^{\propto_6} E_m &= \frac{\beta_m S_m I_h}{N} - (\delta_m + \delta) E_m \\ {}_{a}^{c}D_t^{\propto_7} I_m &= \alpha_m E_m - \delta I_m \\ {}_{a}^{c}D_t^{\propto_8} C &= \alpha_h E_h \end{aligned} \quad (3) $$

From theabove fractional order model, graph of susceptible humansover the $50^{th}$ week of outbreak using the same parametersas in Table 1 above can be drawn as under:

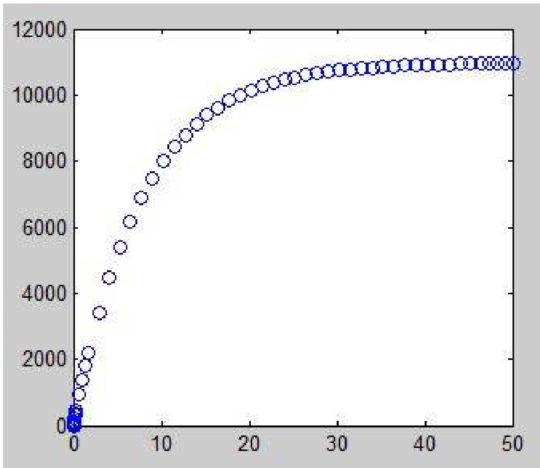


**Figure 2: Susceptible Humans over 50th Weekas per Caputo Fractional Order Model**

**CONFORMABLE FRACTIONAL DERIVATIVE:**

Conformable derivative is the extended form of fractional derivative The concept of Conformable Fractional Derivative has been studied by Khalil [23].Conformable fractional derivative is defined as follows:

Definition 1:The concept of Conformable Fractional Derivative [23], is denoted by $T_{\propto}$ of $f$ for $0 < \propto < 1$.

$$ T_{\propto} f(x) = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon x^{1-\alpha}) - f(x)}{\varepsilon}, \quad x > 0 \quad (4) $$

provided the limit exists, in this case f is called $\alpha$ differentiable on (0; α) for the same, α > 0 also

$$ T_{\alpha}\ f(0) = lim_{x \to 0} T_{\alpha}\ f(x) \quad (5) $$

It has been analyzed by Abdelhakin and Machado that if f is α differentiable in the conformable sense at $x > 0$, then it must be differentiable in the classical sense at $x$ and

$$ T_{\alpha}\ f(x) = x^{1-\alpha} f^{i}(x) $$

is satisfied.

Conformable Fractional Derivative also satisfies the properties provided by Katugampola as under:

- $D^{\propto}[p.f + q.g] = pD^{\propto}[f] + qD^{\propto}[g]$(Linear Property)
- $D^{\propto}[fg] = f.D^{\propto}[g] + g.D^{\propto}[f]$(Product Rule)
- $D^{\propto}[f(g)] = \frac{df}{dt} D^{\propto}[g]$(Chain Rule )

The operator $D^{\alpha}$satisfies

$$ D^{\alpha}[y] = t^{1-\alpha} y \ or \ D^{\alpha} = t^{1-\alpha} D. $$

For 1 < α < 2, $D^{\propto}[y]$ that is anti derivative,$I^{\alpha}$is as follows

$$(D^{\alpha})^{-1} = (D)^{-\alpha} = I^{\alpha} = \int_{\alpha}^{t} \frac{1}{x^{1-\propto}} dx$$

or

$$I^{\alpha}\left[D^{\alpha}[y]\right] = \int_{\alpha}^{t} \frac{x^{1-\alpha}}{x^{1-\alpha}} y' dx = \int_{\alpha}^{t} y' dx = y$$

Definition 2. In article [23], let $\alpha \in (n, n+1)$ and f, be an n- differentiable at t, where t > 0, then the conformable fraction derivative of order α is defined as

$$T_{\propto} f'(t) = \lim_{\varepsilon > 0} \frac{f^{(|x|-1)}t + \varepsilon t^{(|x|-\alpha)} - f^{(|x|-1)}t}{\varepsilon}$$

where [α] is the smallest integer greater than equal to α.

In article [24], Abdeljawad investigated the Chain Rule, Gronwall Inequality, and Integration of Parts formulas, as well as the Laplace transform corresponding to Conformable Fractional Derivative and Integral.

Several analogous properties of classical derivatives, such as Roll's theorem and the Mean Value Theorem, hold true.

The concept of fractional derivative is defined as follows.

In article [23], the Fractional Integral associated with the Conformable Fractional Derivative (derivative) is defined as follows:

$$I_{\propto}^{a}(f)(t) = I_{t}^{a}\left(t^{(\propto-1)} f\right) = \int_{a}^{t} \frac{f(x)}{x^{(\propto-1)}} dx \qquad (6)$$

Khalil et al [23] provided this definition $T_{\alpha}$ $I_{\alpha}^{\alpha}$ (f))(t) = f(t) are inverses of each other, namely$T_{\alpha}$ ($I_{\alpha}^{\alpha}$ (f)) (t) = f(t).

For the t > a where f is continuous in the domain of $T_{\alpha}^{a}$ , we can see that$\alpha \in [0,1]$ and the integral is the usual Riemann improper integral.

Definition 4: In article [25] Katugampola fractional derivative defined by $D^{\alpha}, 0 < \alpha < 1$ is defined as follows:-

$$D^{\alpha} f(t) = lim_{\epsilon \to 0} \frac{f^{(te^{\varepsilon t})} - f(t)}{\epsilon} \; t > 0, \qquad (7)$$

$$D^{\alpha}[f(0)] = lim_{\epsilon \to 0} D^{\alpha}[f(t)].$$

Here, one interesting thing that arises is how a function's conformable fractional derivative f : $[0, \infty] \to R$ is related to minima and maxima of over $[0, \infty]$. The definition given by Kalugampala e-printarxiv:1410.6535, and studied in detail by Anderson Unless [25] is similar to conformable derivative based on the concept of limit rather than a fractional integral.

Here we consider the classical SEIR-SEI model in terms of Conformable Fractional Derivative.

$$t^{(1-\propto)} \frac{dS_h}{dt} = -\beta_h S_h I_v$$

$$t^{(1-\propto)} \frac{dE_h}{dt} = \beta_h S_h I_v - \alpha_h E_h$$

$$t^{(1-\propto)} \frac{dI_h}{dt} = \alpha_h E_h - \gamma I_h$$

$$t^{(1-\propto)} \frac{dR_h}{dt} = \gamma I_h$$

$$t^{(1-\propto)} \frac{dS_m}{dt} = \delta - \frac{\beta_m S_m I_h}{N} - \delta S_m \qquad (8)$$

$$t^{(1-\propto)} \frac{dE_m}{dt} = \frac{\beta_m S_m I_h}{N} - (\sigma + \delta) E_m$$

$$t^{(1-\propto)} \frac{dI_m}{dt} = \alpha_m E_m - \delta I_m$$

$$t^{(1-\propto)} \frac{dC}{dt} = \alpha_h E_h$$

From the above Fractional Order Model, the graph of susceptible humans over the $50^{th}$ week of outbreak using same parameter from Table1reflects as under:

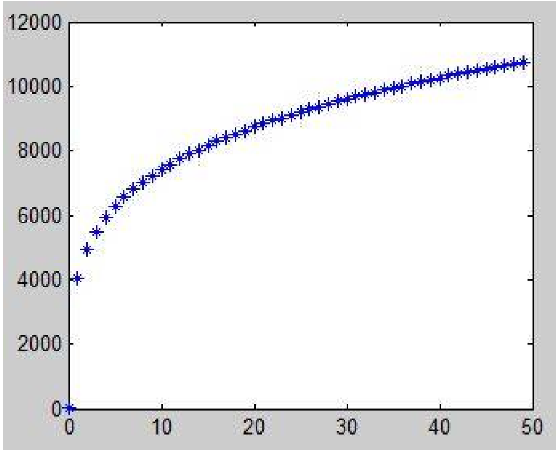


**Figure 3: Susceptible Humans over 50$^{th}$ weekas per Conformable Fractional Order Model**

Below is the comparison of graphs of susceptible humans based on Integer Order Model, Caputo Fractional Order Model and Conformable Fractional Order Model.

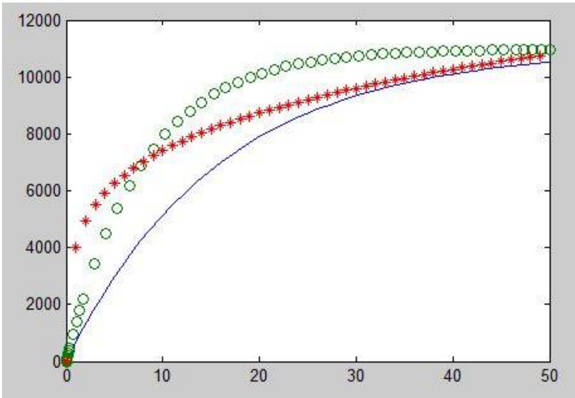


**Figure 4: Comparison of Graphs: Classical order Model, Caputo Model and Conformable Model.**

## CONCLUSION

In this paper, we have replaced Classical Derivative by Caputo Derivative and Conformable Derivative and have studied the effects of this change which is visible in the graph. It is clear that in the time duration of 40-50 weeks, the numerical difference of susceptible persons as per the Conformable Model is almost equal to that of based on the Caputo and Classical Models.

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