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ORIGINAL ARTICLE **OPEN ACCESS**

Impact of slip velocity and stenosis morphology on blood flow through an atherosclerotic artery segment using the Bingham-Plastic fluid model

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ABSTRACT
The influence of slip velocity and stenosis form on the flow of blood through a stenosed artery segment is estimated using a mathematical model of blood in this theoretical study. The presence of red cells (erythrocytes) in plasma, which exhibits blood's non-Newtonian features, is thought to be explained by Bingham plastic fluid model of the blood. The polynomial form of the stenotic geometry is used to solve the coupled differential equations that govern fluid flow, as well as the constitutive equation (in one dimension) for Bingham-plastic fluid. Because of the permeability of the vessel wall, the slip velocity in the stenosed artery wall is given special consideration. The effect of stenosis height, stenosis shape parameter, slip velocity, yield stress on the wall, shear stress, resistance to flow, axial velocity of blood, and volumetric flow rate is investigated using both analytical and numerical techniques. For clinical examination, the minimum flow rate at a site in the stenotic area is found.
Keywords: *Stenosis, Bingham-Plastic fluid model, slip velocity, resistance to flow, stenosis shape parameter, wall shear stress*

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INTRODUCTION

Atherosclerosis is a metabolic disease of the arteries that is directly linked to blood flow and its dynamic activity. The aberrant narrowing of the open space in the artery as a result of cholesterol-rich fatty or lipid material deposition and growth of associated tissues causes major circulation problems. Stenosis is the medical word for an abnormal and unnatural increase in the thickness of the artery wall in various parts of the cardiovascular system. Hypertension, ischemia, and heart failure are all caused by it restricting blood flow in the system. Although the exact cause and mechanism of stenosis formation in the arterial lumen is unknown from a physiology/pathology standpoint, it is known that rheological and hemodynamic factors may play a role in the basic understanding, diagnosis, and treatment of many cardiovascular, cerebrovascular, and arterial diseases. On the assumption that the fluid representing blood is Newtonian, single phase homogenous viscous fluid, and incompressible, a large number of analytical and experimental investigations pertinent to the topic under discussion have been conducted. Blood, which is a suspension of erythrocytes (red cells) in plasma, has been shown in numerous experimental and analytical studies to exhibit non-Newtonian behaviour in micro capillaries at low shear rates. The problem of blood flow via a stenosed portion of an artery was studied by Nanda et al. [5], who used the Herschel-Bulkley fluid model to describe the rheology of blood. The usual experimental form accurately depicted the stenosis' bell-shaped geometry. The effect of modest stenosis on streaming blood flow in an irregular axi-symmetric artery with oscillating pressure gradient was investigated by Jain et al. [2]. Jain et. al. [1] created a mathematical model for MHD flow in a stenosed artery using a smooth cosine curve as the stenotic geometry. Bhatnagar et al.[3] gave a theoretical analysis of blood flow based on the Bingham-Plastic fluid model of blood in polynomial form in the stenotic zone. No slip condition at the restricted walls was taken in the majority of earlier investigations. The occurrence of a slip velocity discontinuity at the artery wall has been postulated by Chaturani et. al. [4].

It allows us to investigate the problem of blood flow through a stenosed artery segment, where the rheology of blood is characterised by the Bingham- Plastic fluid model with slip at the artery wall. The general polynomial curve with variable stenosis parameter depicts the stenosis that is assumed to manifest in the arterial segment. The study has become more general and practical as a result of these factors.
Numerical computations of observable flow variables with greater rheological and physiological significance are used to perform a comprehensive quantitative analysis. Their graphical representations are offered at the conclusion of the work, along with relevant scientific considerations. Finally, the applicability of the current mathematical model is demonstrated through comparisons with existing results.

MODEL DESCRIPTION

In this study, we look at the axially-symmetric flow of blood through a circular-cross-section artery with a stenosis. The geometry of the stenosis as it is assumed to present itself in the arterial segment (assumed to be a rigid circular tube) by

$$\frac{R(z)}{R_0} = \begin{cases} 1 - A\left[l_0^{m-1}(z-d) - (z-d)^m\right], & d \le z \le d + l_0 \\ 1, & otherwise \end{cases} \quad (1)$$

$R(z)$: radius of the stenosed portion of the arterial segment

R_0 : radius of the artery outside the stenosis

l_0 : length of stenosis

m : stenotic parameter $2 \le m \le 11$ [$m = 2$ gives radially symmetric stenosis] d : indicates the location of the stenosis

δ : Maximum height of the stenosis at $z = d + \frac{l_0}{m^{1/(m-1)}}$ subject to the condition $\frac{\delta}{R_0} << 1$

and the parameter A is given by $A = \frac{\delta}{R_0 l_0^{m-1}} \cdot \frac{m^{m/(m-1)}}{m-1}$ (2)

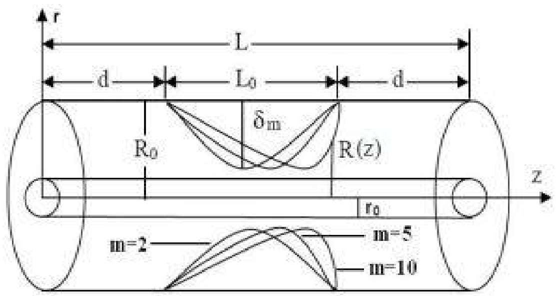

Fig 1. Geometry of the stenosed arterial segment

Blood is thought to flow in a steady, laminar, and fully developed manner. The constitutive equation for Bingham-Plastic fluid in one dimension is

$$-\frac{du}{dr} = f(\tau) = \begin{cases} \frac{\tau - \tau_0}{k}, \tau \ge \tau_0 \\ 0, \quad \tau < \tau_0 \end{cases}$$ Bhatnagar et. a*l*.[3](3)

u : axial velocity of blood τ : shearing stress τ_0 : yield stress k : coefficient of viscosity of blood.

(Here the artery is taken as a rigid circular tube). A number of theoretical and experimental research on blood and blood flow through arteries (in the sick state) have suggested that the Newtonian model of blood may be considered if the shear rate of blood is high enough. However, because the shear rate of blood is relatively low near the centre of the artery (modelled as a rigid circular tube), the non-Newtonian model of blood is more acceptable (cf. Mishra et al. [8]).

The equation governing the flow of blood may be written as $-\frac{dp}{dz}=\frac{1}{r}\frac{d(r\tau)}{dr}$ (Mishra et. al.[7]) (4)

(z,r) : coordinates of a point with z along the axis of the arterial segment and r measured along the normal to it

p :pressure at any point

$-\frac{dp}{dz}$: pressure gradient

The boundary conditions pertaining to the problem are

$u=u_s \ at \ r=R(z)$ (slip velocity condition) (5)

τ is finite at $r=0$ (regularity condition) (6)

ANALYTICAL SOLUTION

Integrating equation (4) and applying boundary condition (6) , $\tau=-\frac{1}{2}r\frac{dp}{dz}$.

If r_0 = radius of the core region then the yield stress (τ_0) in the core region is given by

$\tau_0=-\frac{1}{2}r_0\frac{dp}{dz}\Rightarrow\tau-\tau_0=-\frac{1}{2}(r-r_0)\frac{dp}{dz}$ (7)

The wall shear stress (τ_R) is given by $\tau_R=-\frac{1}{2}R\frac{dp}{dz}$ (8)

Blood will flow within the circular tube if $\tau\geq\tau_0$ $i.e. if$ $r\geq r_0$. Blood will not flow if $r<r_0$.

As $u(r)$ represents the axial velocity of blood so for $r<r_0, \frac{du_r}{dr}=0$

Integrating, $u(r)=const=u_0$ = velocity of blood in the core region.

If the pressure gradient $-\frac{dp}{dz}=P(z)=P$ (the pressure gradient) then for the region $r_0\leq r\leq R(z)$,

(7) becomes $\tau-\tau_0=\frac{1}{2}(r-r_0)P$ and the equation (3) takes the form $\frac{du}{dr}=-\frac{P}{2k}(r-r_0)$ (9)

Integrating between the limits r & R,

$$\int_r^R du=-\frac{P}{2k}\int_r^R (r-r_0)dr\Rightarrow u(R)-u(r)=-\frac{P}{4k}\left[(R-r_0)^2-(r-r_0)^2\right],\ r_0\leq r\leq R(z) \quad (10)$$

Following the boundary condition (5), $u(r)=u_s+\frac{P}{4k}\left[(R-r_0)^2-(r-r_0)^2\right]$ (11)

To get the expression for core velocity (u_0), $u_0=u_s+\frac{P}{4k}(R-r_0)^2$ (12)

The volumetric flow rate is defined as $Q=\int_0^R 2\pi\, r\, u(r)du=\int_0^{r_0} 2\pi\, r\, u(r)du+\int_{r_0}^R 2\pi\, r\, u(r)du$ (13)

As from (9) , $du=-\frac{P}{2k}(r-r_0)dr$ (14)

so using the value of $u(r)$ from (11) , du in (14) and after integration one gets

$$Q=\pi R^2u_s+\frac{\pi PR^4}{8k}\left(1+\frac{2}{3}t+\frac{1}{3}t^2\right)(1-t)^2 \quad (15)$$

where $t=\frac{r_0}{R}=\frac{\tau_0}{\tau_R}$ by (7) and (8)

Usually yield stress (τ_0) is much less than the wall shear stress (τ_R) so $\frac{\tau_0}{\tau_R} << 1,$ neglecting higher powers of $t,$ (15) becomes $Q = \pi R^2 u_s + \frac{\pi P R^4}{8k}\left(1 - \frac{4}{3}t\right)$ (16)

As $t = \frac{\tau_0}{\tau_R} = \frac{\tau_0}{(R/2)P}$ so from (16) the pressure gradient is given by

$$-\frac{dp}{dz} = P = \frac{8k}{\pi R^4}\left(Q - \pi R^2 u_s\right) + \frac{8}{3}\frac{\tau_0}{R} \quad (17)$$

$$-dp = \frac{8kQ}{\pi R_0^{\ 4}} \frac{dz}{(R/R_0)^4} - 8ku_s R_0^{\ 2} \frac{dz}{(R/R_0)^2} + \frac{8}{3}\frac{\tau_0}{R_0}\frac{dz}{((R/R_0)}$$ where $\frac{R}{R_0}$ is given by (1)

Integrating throughout the length (*l*) of the artery where $p = P_1$ at $z = 0$ and $p = P_2$ at $z = l$

$$P_1 - P_2 = \frac{8kQ}{\pi R_0^{\ 4}}\int_0^l \frac{dz}{(R/R_0)^4} - 8ku_s R_0^{\ 2}\int_0^l \frac{dz}{(R/R_0)^2} + \frac{8}{3}\frac{\tau_0}{R_0}\int_0^l \frac{dz}{(R/R_0)}$$

Thus, the resistance to flow $\lambda_R = \frac{P_1 - P_2}{k} =$

$$\frac{8k}{\pi R_0^{\ 4}}\left[l - l_0 + \int_d^{d+l_0} \frac{dz}{(R/R_0)^4}\right] - \frac{8ku_s}{QR_0^{\ 2}}\left[l - l_0 + \int_d^{d+l_0} \frac{dz}{(R/R_0)^2}\right] + \frac{8}{3}\frac{\tau_0}{QR_0}\left[l - l_0 + \int_d^{d+l_0} \frac{dz}{(R/R_0)}\right] \quad (18)$$

We use the non-dimensional variables as

$$l_1 = 1 - \frac{l_0}{l} + \frac{1}{l}\int_d^{d+l_0} \frac{dz}{(R/R_0)^4}, \quad l_2 = 1 - \frac{l_0}{l} + \frac{1}{l}\int_d^{d+l_0} \frac{dz}{(R/R_0)^2}, \quad l_3 = 1 - \frac{l_0}{l} + \frac{1}{l}\int_d^{d+l_0} \frac{dz}{(R/R_0)}$$

The non-dimensional form of the resistance to flow (λ) and the wall shear stress (τ) are given by

$$\lambda = \frac{3Q.l_1 - 3\pi k u_s.R_0^{\ 2}.l_2 + \pi R_0^{\ 3}.\tau_0.l_3}{3kQ + \pi R_0^{\ 3}.\tau_0} \quad (19)$$

$$\tau = \frac{3k\left[Q - \pi.R_0^{\ 2}(R/R_0)^2 u_s\right] + \pi R_0^{\ 3}(R/R_0)^3 \tau_0}{(R/R_0)^3\left[3kQ + \pi R_0^{\ 3}\tau_0\right]} \quad (20)$$

NUMERICAL RESULTS AND DISCUSSIONS

It is necessary to offer a graphical presentation based on numerical computation in order to acquire a quantitative estimate of the influence of various physical and rheological parameters involved in the analysis. The purpose of the presentation is to assess the model's validity. The following values of the parameters were selected from standard literature for the current computational investigation.
Computations are carried out taking the values of the following parameters as constant

R_0=1.5, l=5, μ = 0.000345 p.a.s., τ_0=0.02 n/m^2 and $\frac{dp}{dz} = 1$

The analysis corresponds to stenosis shape parameter m = 2,4,6,8 & 11 and stenosis height $\frac{\delta}{R_0}$ = 0.2(mild stenosis), 0.4 and 0.6(moderate stenosis) and 0.8(severe stenosis). Actually the degree of stenosis depends upon stenosis height. Through experimental observations it is established that m lies between 2 & 11. The corresponding results Power Law Fluid Model can be obtained taking $\tau_0 = 0$. Also $m = 2$ gives the results for the case of symmetric stenosis.

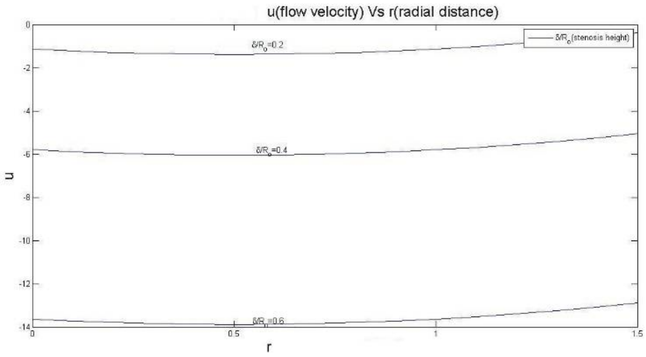

Fig.2 Variation of flow velocity along radial distance for different values of stenotic height ($\frac{\partial}{R_0}$)

An attempt is made through Fig.2 to show the variation of flow velocity of blood with the radial distance for stenotic heights 0.2, 0.4 and 0.6. It shows that the flow velocity decreases with the increase of stenotic height.

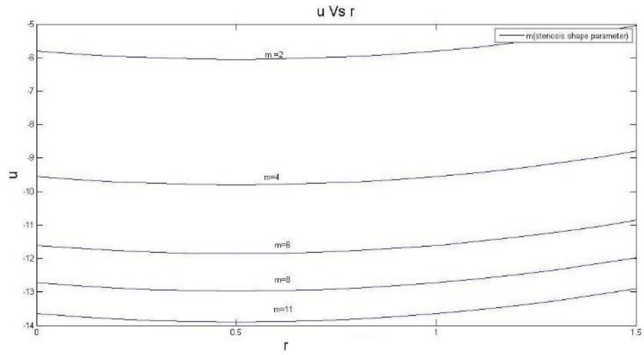

Fig.3 Variation of flow velocity along radial distance for different values of stenosis shape parameter (m)

Fig.3 depicts the variation of flow velocity with the radial distance for different values of the stenosis shape parameter (m), m=2 is the condition that the stenosis is symmetric. Naturally the blood velocity decreases with the increase in value of m.

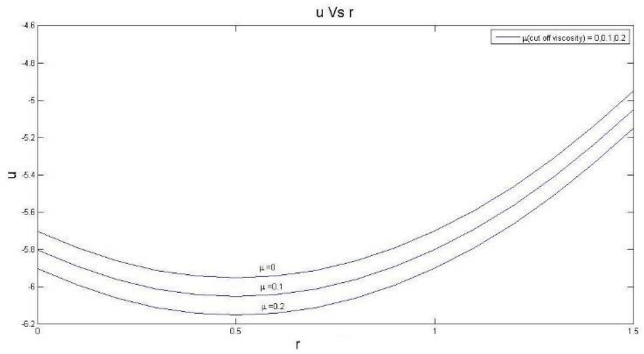

Fig.4 Variation of flow velocity along radial distance for different values of viscosity coefficient (μ)

Fig.4 illustrates the variation of flow velocity with the radial distance for μ (viscosity coefficient) = 0, 0.1 and 0.2

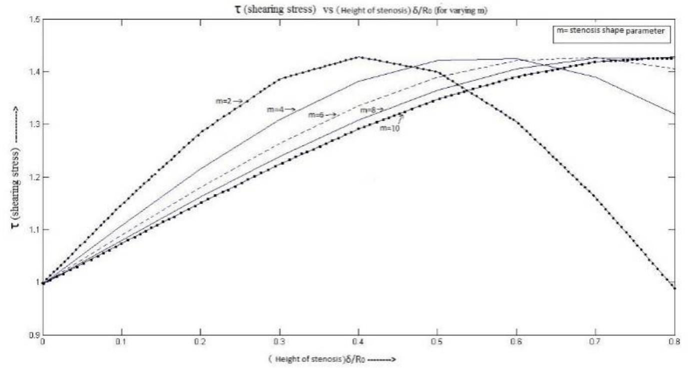

Fig.5 Variation of shear stress with stenotic height for different values of stenosis shape parameter (m)

Fig.5 exhibits the variation of shear stress with stenotic height considering m= 2,4,6,8 and 10

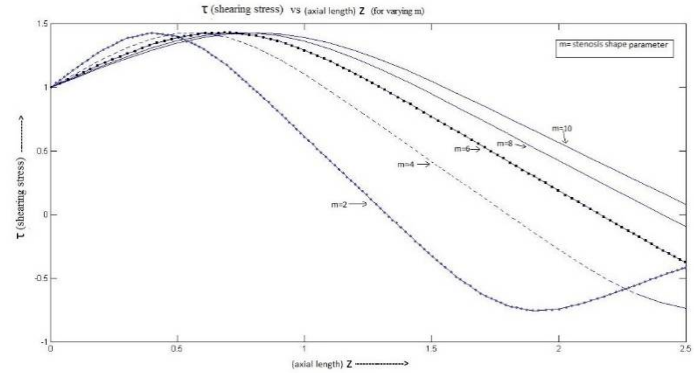

Fig.6 Variation of shear stress with axial distance (z) for different values of stenosis shape parameter (m)

In Fig.6, the variation of shear stress (τ) with z (measured along the axis of the artery) is presented considering m = 2,4,6,8,10.

Some investigators like Chaturani et.al.[4], Brunn[6] have rightly suggested that due to the permeability of the vessel wall, no slip condition at the wall is merely a simplification. In this investigation, the likely presence of slip velocity at the flow boundaries is given due emphasis.

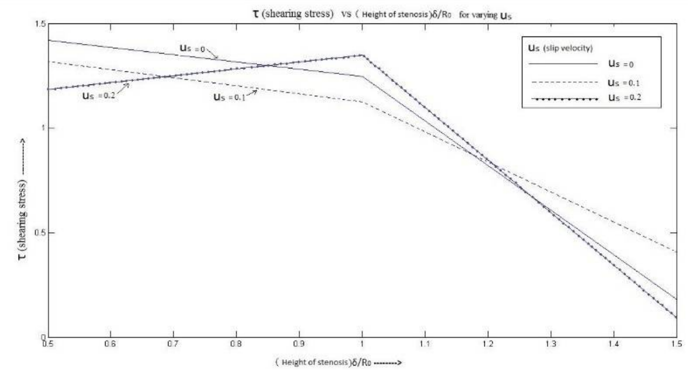

Fig.7 Variation of shear stress with stenotic height for different values of slip velocity (u_s)

Fig.7 exhibits the variation of shearing stress with stenotic height considering the slip velocity u_s = 0 ,0.1and 0.2. It is experimentally established that the shape parameter m lies between 2 and 11.

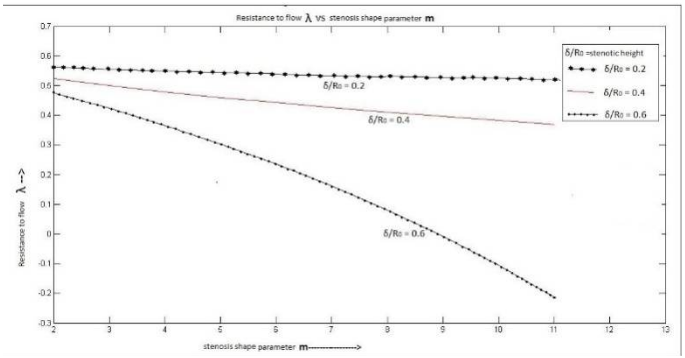

Fig.8 Variation of resistance to flow (λ) with stenosis shape parameter (m) for different values of stenotic height $(\frac{\partial}{R_0})$

Fig.8 shows the effect of the shape parameter on the flow resistance varying the stenotic height $\frac{\delta}{R_0}$=0.2, 0.4 and 0.6.

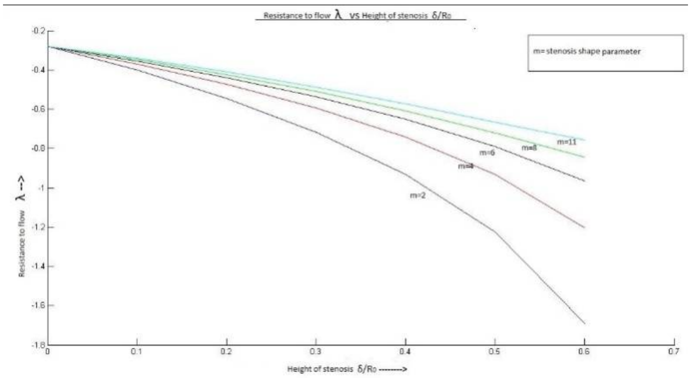

Fig.9 Variation of resistance to flow (λ) with stenotic height $(\frac{\partial}{R_0})$ for different values of stenosis shape parameter (m)

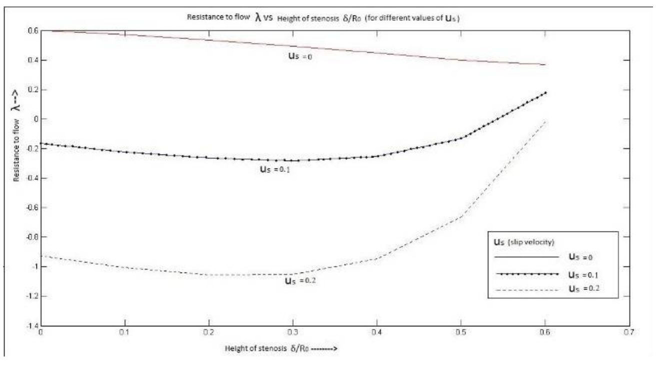

Fig.10 Variation of resistance to flow (λ) with stenotic height $(\frac{\partial}{R_0})$ for different values of slip velocity (u_s)

Fig.9 & Fig.10 depict the variation of flow resistance due to the variation of stenotic height for different values of m and u_s.

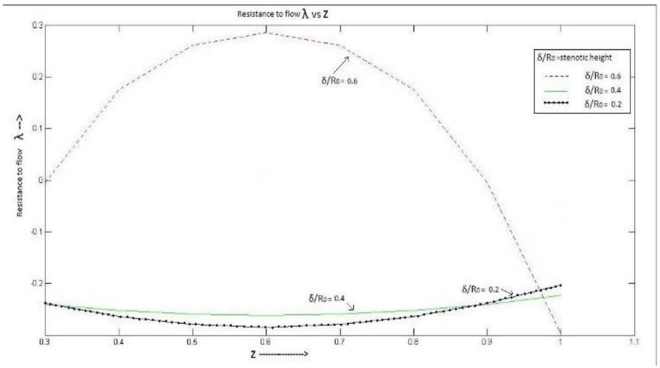

Fig.11 Variation of resistance to flow (λ) along axial distance (z) at different values of stenotic height ($\frac{\delta}{R_0}$)

The variation of flow resistance with z(measured along the axis) is presented in Fig.11 considering the stenotic height $\frac{\delta}{R_0}$ = 0.2,0.4 and 0.6.

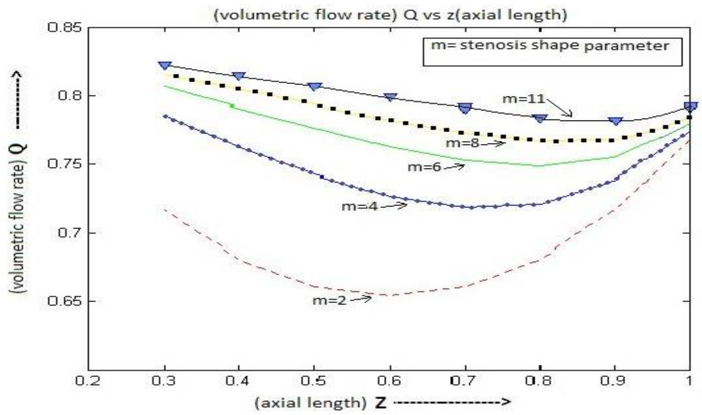

Fig.12 Variation of volumetric flow rate (Q) along axial distance (z) for different values of stenosis shape parameter (m)

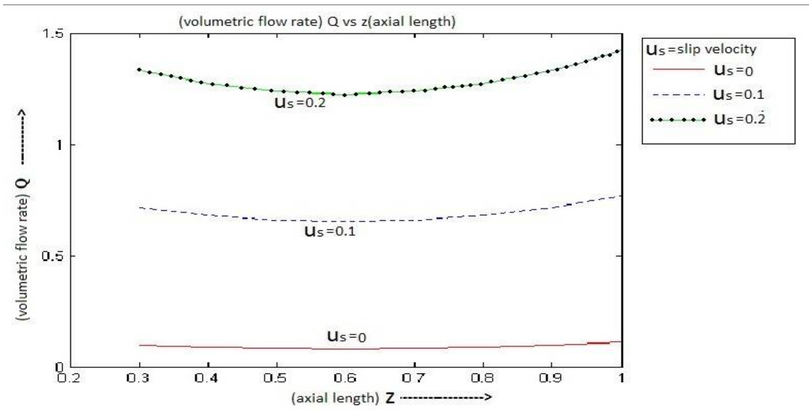

Fig.13 Variation of volumetric flow rate (Q) along axial distance (z) for different values of slip velocity (u_s)

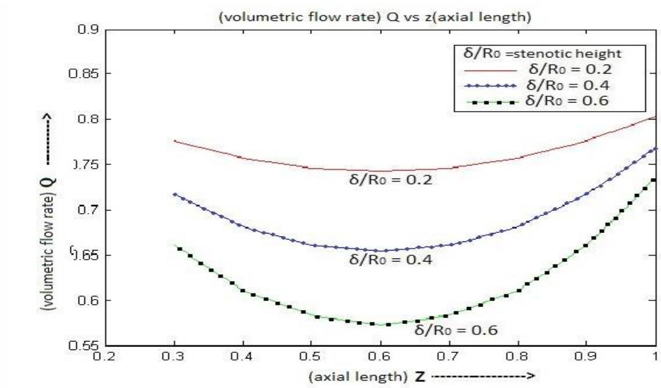

Fig.14 Variation of volumetric flow rate (Q) along axial distance (z) for different values of stenotic height ($\frac{\delta}{R_0}$)

Fig.12, 13 and 14 illustrate the likely variation of the volumetric flow rate(Q) with axial distance(z) considering the change of m(stenotic shape parameter), u_s(slip velocity) and $\frac{\delta}{R_0}$(stenotic height) all in their admissible range.

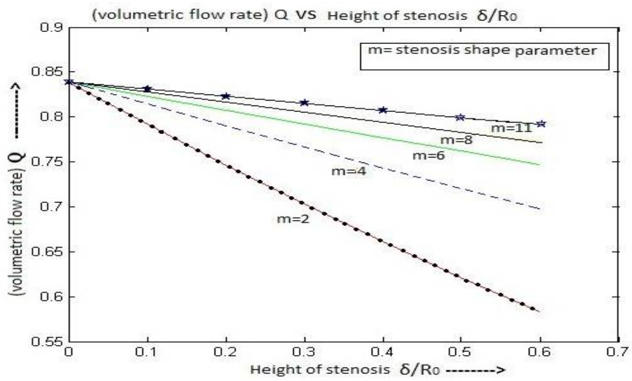

Fig.15 Variation of volumetric flow rate (Q) with stenotic height ($\frac{\partial}{R_0}$) for different values of stenosis shape parameter (m)

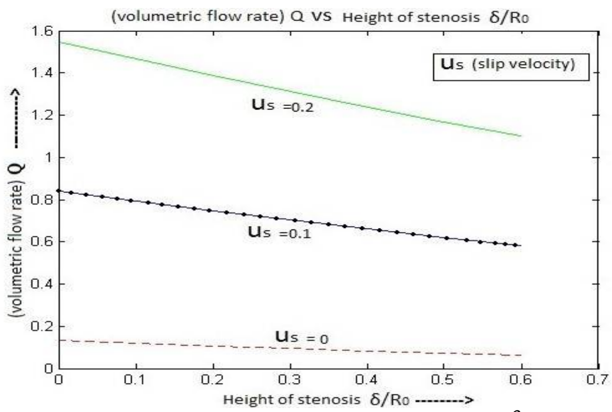

Fig.16 Variation of volumetric flow rate (Q) with stenotic height ($\frac{\partial}{R_0}$) for different values of slip velocity (u_s)

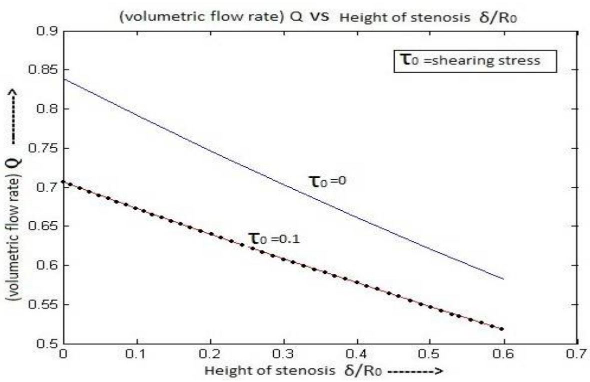

Fig.17 Variation of volumetric flow rate (Q) with stenotic height ($\frac{\partial}{R_0}$) for different values of shearing stress (τ_0)

The variation of the volumetric flow rate (Q) with stenotic height $\frac{\delta}{R_0}$ are presented in Figs.15, 16 & 17 paying attention on stenotic shape parameter, slip velocity and τ_0 (shearing stress = 0 and 0.1)

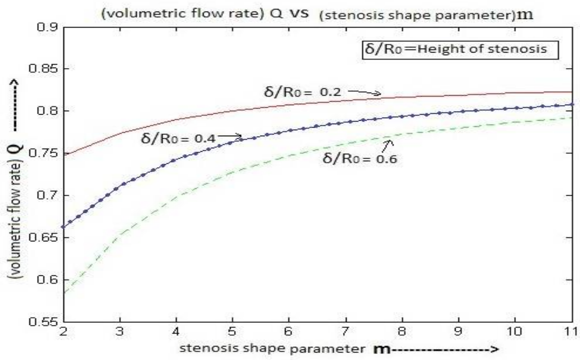

Fig.18 Variation of volumetric flow rate (Q) with stenosis shape parameter (m) for different values of stenotic height ($\frac{\delta}{R_0}$)

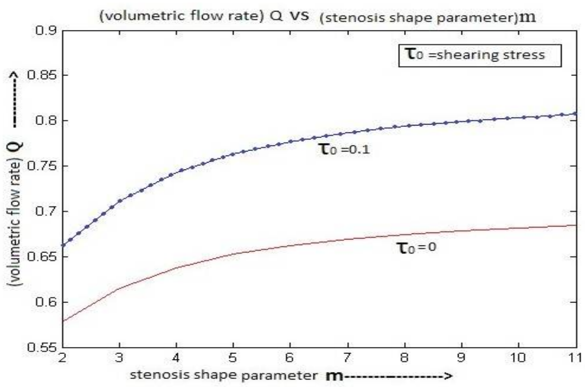

Fig.19 Variation of volumetric flow rate (Q) with stenosis shape parameter (m) for different values of shear stress(τ_0)

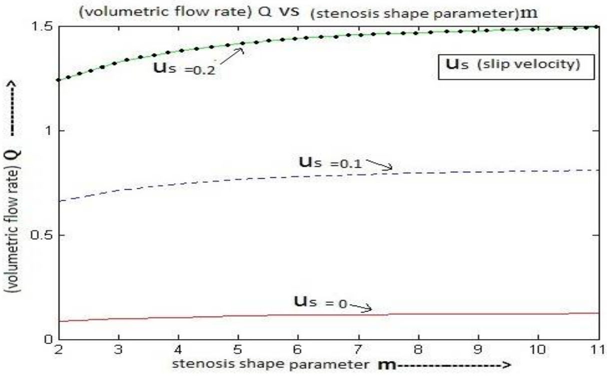

Fig.20 Variation of volumetric flow rate (Q) with stenosis shape parameter (m) for different values of slip velocity (u_s)

The last three figures in the dissertation(Figs18-20) demonstrate the variation of the volumetric flow rate(Q) with m ($\frac{\delta}{R_0}$ =0.2 ,0.4,0.6), with m(τ_0= 0, 0.1)and with m(μ_s= 0.1, 0.2)

CONCLUSION

In this investigation, the rheology of blood flowing through a narrow artery segment with a stenosis that is symmetrical about the axis but non-symmetrical in radial dimensions was theoretically examined using the Bingham Plastic fluid model for blood. The flow and flow properties are influenced qualitatively and quantitatively by the stenosis shape parameter, stenotic height, slip velocity, viscosity coefficient, shear, and yield stresses. This mathematical analysis allows for the estimation of the importance of the rheological and physical factors in the analysis, as well as the determination of which of the parameters plays the most important part in the creation and progression of arterial disease. It is envisaged that the results of the analysis will assist clinicians in determining the stenotic range, crucial location, and severity of the disease under the scope of a single research using the Bingham Plastic fluid model of blood. It may aid them in making difficult medical or surgical treatment decisions. Incorporating further rheological and physical characteristics, as well as overcoming the constraints imposed in this research, would encourage more thorough investigation.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest

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