Bulletin of Environment, Pharmacology and Life Sciences
Bull. Env.Pharmacol. Life Sci., Vol 11[2] January 2022 :33-46

Online ISSN 2277-1808
Journal's URL:http://www.bepls.com
CODEN: BEPLAD
ORIGINAL ARTICLE **OPEN ACCESS**

Combined Array Approach for 16-Run Non-equivalent Designs

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ABSTRACT
Variation in product quality is inevitable due to differences in Material, Man power, Money, Machine and Methods. To reduce this variation, improve product qualityand design robustness,[1] introduced robust parameter design technique using product arrays. However, the prohibitive cost due to the excessive number of runs ina product array led to the introduction of combined arrays, wherein the control and noise factors are combined in a single array. In this paper, we studied the effect of a number of variables on yield of paddy crop using the concept of combined arrays for 16-run non-equivalent designs with two controllable factors viz. variety of fertilizers (A), type of seeds (B) and three uncontrollable factors viz. temperature (r), humidity (s) and rainfall (t).
Key Words: Robust Parameter Design, Combined Arrays, Control Factors, Noise Factors, D-efficiency, Interaction Graphs.

Received 15 .12.2021 Revised 20.01.2022 Accepted 31.01.2022

INTRODUCTION

In agricultural sciences, designing an experiment is an imminent part of every research. The data generated from these designed experiments show a lot of variability. The variability may be wanted and desirable, or unwanted and undesirable, but controllable as in it very well may be represented. There is likewise some more variability, which isundesirable, unwanted and uncontrollable. The justification for its presence is obscure. For example, the experimental units (plots) exposed to a similar treatment bring about various perceptions and subsequently make variability. The variation might arise because of variable fertility/soil moisture/soil depth, etc. of the two plots.In fractional factorial experiments, we can change two or more things at a time, where the complete set of factorial treatment combinations is to be included in the test, the fractional factorial design, as the name implies, includes only a fraction of the complete set of the factorial treatment combinations. Combined Array Approach is very useful to achieve the optimal settings of the control factors in designed experiments at which the variation in response induced by noise factors is minimum. When we have controllable factors like fertilizers and type of seeds and uncontrollable factors like weather parameters viz. temperature, relative humidity in morning and evening, total rainfall etc. in designed experiments then with the help of the combined array approach using a 16-run non-regular design we can reduce the number of runs in contrast with the product array approach as well as allows more flexibility in the selection of estimable effects of interest. Robust Parameter Design (RPD) introduced by [1] is an off-line quality improvement methodology which consists in finding the optimal settings of the control factors at which the variations in response caused by the uncontrollable noise factors are minimum. Control factors can be controlled in an experiment and also in a real application of the product such as cycle time, type of the material used for manufacturing, the equipment settings of the product etc. Noise factors on the other hand can be controlled in an experiment but are difficult or impossible or too expensive to control in the actual process or usage of the product such as humidity conditions, ambient temperature etc.
To implement RPD technique, [1] advocated the use of product arrays, where an inner array containing the settings of control factors is crossed with an outer array containing the noise factors and their settings. The crossing of two orthogonal arrays in a product array often results in a large number of runs. Also, the models supported by the product arrays allow the estimation of all interactions between the control and noise factors but no interactions among the control factors or among the uncontrollable factors can be estimated. Several statistical alternatives have been suggested to overcome the drawbacks

of the product array technique.[2], [3], [4], [5], [6],[7], [8], [9], [10] and [11] suggested the use of combined arrays, wherein the control factors and noise factors are combined in a single array. Combined arrays apart from reduction in run size allow more flexibility in the selection of effects of interest so that the experimental budget can be used to fit models more refined than the main effects only models frequently used in Taguchi's loss model approach. An excellent review of the robust parameter technique is made by [12] and [13]. [14]Compared Taguchi's product array with a combined array. An experiment was conducted for preparation of super absorbent composites with maximum water absorption characteristics and enhanced stability and moisture absorption behaviour in plant growth media to achieve maximum and fast rate of absorbency utilizing minimum possible concentration of monomer, cross linker and alkali [15].

The combined arrays discussed by the authors mentioned above pertain to the use of regular fractional factorial designs. In this paper, we have exploited the non-regular structure of the 16-run non-equivalent designs and have generated non-regular combined arrays using them. Our models always contain first order effects in the control and noise factors and if possible, all interactions between control and noise factors (C × N). Owing to availability of degrees of freedom we sometimes also include control × control (C × C) interactions. We have also drawn non-isomorphic interaction graphs corresponding to the selected designs. These interaction graphs enable engineers, scientists and non-statisticians in choosing the column allocations of various designs so that all main effects and required two-factor interactions can be estimated with high efficiency. Let $X_1, X_2, \ldots X_r$ be r control factors, and $Z_1, Z_2, \ldots, Z_s$ be s noise factors. The objectives of the study are:

1) To estimate the main effects of all the control factors and noise factors.
2) To estimate the C x N interactions.
3) To estimate, if possible, (depending upon degrees of freedom) the C × C interactions.

MATERIAL AND METHODS

Supporting Models

Let y denote a quality characteristic associated with a product. Then in order to meet the first two objectives, the supporting model, by keeping the origin at (0,0), would be:

$$y = \Sigma\beta_i x_i + \Sigma\gamma_j z_j + \Sigma\Sigma\delta_{ij} x_i z_j + \epsilon \quad (2.1)$$

To meet the third objective, i.e. estimating the C x C interactions, the corresponding model would be:

$$y = \sum \beta_i x_i + \sum \gamma_j z_j + \sum\sum \delta_{ij} x_i z_j + \sum\sum \beta_{ii'} x_i x_{i'} + \epsilon \quad (2.2)$$

Efficiency Criterion

Among various optimality criterion discussed in literature for comparing designs, the D-optimality criterion is one of the most popular criterion which aims at maximizing $\det(X'X)$, the determinant of the $X'X$matrix. A design which is D-optimal will yield the highest D-efficiency value. We have used the following D-criterion for measuring the overall efficiency for estimating a collection of effects

$$\text{D-efficiency} = |X'X|^{1/k} \quad (3.1)$$

where $X = [x_1/||x_1||, \ldots, x_k/||x_k||]$; and x_i is the coefficient vector of the i^{th} effect. Since the columns of X are standardized, (3.1) achieves its maximum if and only if the x_i's are orthogonal to each other. The vector 1 is not included in X.

To find the efficiency of each individual effect, we have used the following D_s criterion: -

$$\frac{\left\{x_i' x_i - x_i' X_{(i)} \left(X_{(i)}' X_{(i)}\right)^{-1} X_{(i)}' X_{(i)}\right\}}{x_i' x_i} \quad (3.2)$$

where $X_{(i)}$ is obtained from X by deleting x_i. (3.2) attains its upper bound 1 if and only if x_i is orthogonal to the other columns in X.

Algorithm Used for the Combined Array Approach

In Taguchi's RPD, one attempts to achieve the optimal settings of the control factors at which the variation in response induced by noise factors is minimum. This is accomplished by exploiting the interactions between control factors and noise factors. The structure of these interactions determines the nature of non-homogeneity of the process variance that characterizes the parameter design problem. Noise × noise interactions hardly play any role in making a product's performance insensitive to noise factors. The presence of large control × control interactions is highly undesirable, soevery attempt is made to reduce their number by judicious choice of the quality characteristics.

Thus, giving top most priority to the estimation of control × noise interactions, we now discuss the procedure followed in the combined array approach:

(a) Choose p columns from the totality of n-1 columns of thedesign and consider all possible non-isomorphic designs corresponding to these p columns. Two designs are said to be isomorphic if one can be obtained from the other by permuting rows or columns or changing the signs of rows or columns.

(b) For each design, allocate the control factors and noise factors top columns.

(c) Write the appropriate model by considering the required set of C x N interactions and C x C interactions (depending upon run-size).

(d) For all possible choices of the control and noise factors find the D-efficiency for the whole design and D_s values for the various effects.

(e) Compare the D-efficiency of all the designs obtained and retain all the designs with maximum D-efficiency.

(f) Sort the D_s values of these designs on the basis of C x N interactions and take all designs for which it is maximum.

(g) Now sort the D_s values of the designs retained on the basis of C x C interactions and take all designs for which it is minimum.

(h) Finally sort these designs on the basis of the D_s values for control factors and noise factors and take the designs for which it is maximum.

(i) Following the technique of drawing non-isomorphic interaction graphs given by Wu and Chen [9], then draw non-isomorphic interaction graphs for each of these designs. An interaction graph is the graphical representation of allocation of the main effects (m.e.) and two-factor interactions (2fi.) to the columns of the design. The nodes of interaction graph denote main effects and the edge joining two nodes denotes the two-factor interaction between them.

Results and Discussion

Hall's 16-Run Non-Equivalent Designs

[16]Discovered that there are exactly five non-isomorphic Hadamard matrices of order 16. A Hadamard matrix of order n is an n x n matrix with elements + l and -l such that H'H = nI. Two Hadamard matrices are isomorphic if one can be obtained from the other by permuting rows or columns or changing the signs of rows or columns. When $n = 2^k$, there is a special type of Hadamard matrix whose columns form an elementary Abelian group under the operation of multiplication. This type of Hadamard matrix is also known as the regular two-level fractional factorial design. Hadamard matrices whose columns do not form an Abelian group are called non-regular Hadamard matrices. Thus, all the Hadamard matrices with order $n \neq 2^k$ are non-regular.

[16]Called his five matrices Classes I, II, III, IV and V.Following [17], here we shall call them as HI6-1, HI6...., and HI6-V. Among these five matrices the first matrix HI6-I is the regular 2^4 factorial design and the rest four are non-regular designs that are shown in Table 1. The basic distinction between these two types of designs is the presence of partial aliasing of effects. For regular designs,any two effects are either orthogonal or fully aliased. For non-regular designs some effects are partially aliased, that is, the aliasing coefficient is between +1 and -1.

Table 1: Hall's 16-Run Non-Regular Orthogonal Designs

H16-II																H16-III														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	-	-	-	-	-	-	-	-		+	+	+	+	+	+	+	-	-	-	-	-	-	-	-
+	+	+	-	-	-	-	+	+	+	+	-	-	-	-		+	+	+	-	-	-	-	+	+	+	+	-	-	-	-
+	+	+	-	-	-	-	-	-	-	-	+	+	+	+		+	+	+	-	-	-	-	-	-	-	-	+	+	+	+
+	-	-	+	+	-	-	+	+	-	-	+	+	-	-		+	-	-	+	+	-	-	+	+	-	-	+	+	-	-
+	-	-	+	+	-	-	-	-	+	+	-	-	+	+		+	-	-	+	+	-	-	-	-	+	+	-	-	+	+
+	-	-	-	-	+	-	+	+	-	-	-	-	+	+		+	-	-	-	-	+	-	+	+	-	-	-	-	+	+
+	-	-	-	-	+	+	-	-	+	+	+	+	-	-		+	-	-	-	-	+	+	-	-	+	+	+	+	-	-
-	+	-	+	-	+	+	+	-	+	-	+	-	+	-		-	+	-	+	-	+	+	+	-	+	-	+	-	+	-
-	+	-	+	-	+	-	-	+	-	+	-	+	-	+		-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
-	+	-	-	+	-	-	+	-	+	-	-	+	-	+		-	+	-	-	+	-	-	+	-	-	+	+	-	-	+
-	+	-	-	+	-	+	-	+	-	+	+	-	+	-		-	+	-	-	+	-	+	-	+	+	-	-	+	+	-
-	-	+	+	-	-	+	+	-	-	+	-	+	+	-		-	-	+	+	-	-	+	+	-	-	+	-	+	+	-
-	-	+	+	-	-	+	-	+	+	-	+	-	-	+		-	-	+	+	-	-	+	-	+	+	-	+	-	-	+
-	-	+	-	+	+	-	+	-	-	+	+	-	-	+		-	-	+	-	+	+	-	+	-	+	-	-	+	-	+
-	-	+	-	+	+	-	-	+	+	-	-	+	+	-		-	-	+	-	+	+	-	-	+	-	+	+	-	+	-

H16-IV

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	-	-	-	-	-	-	-	-
+	+	+	-	-	-	-	+	+	+	+	-	-	-	-
+	+	+	-	-	-	-	-	-	-	-	+	+	+	+
+	-	-	+	+	-	-	+	+	-	-	+	+	-	-
+	-	-	+	+	-	-	-	-	+	+	-	-	+	+
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-	+	-	+	-	+	-	+	-	+	-	+	-	+	-
-	+	-	+	-	-	+	+	-	-	+	-	+	-	+
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-	+	-	-	+	-	+	-	+	+	-	-	+	+	-
-	-	+	+	-	+	-	-	+	-	+	-	+	-	-
-	-	+	+	-	-	+	-	+	+	-	+	-	-	+
-	-	+	-	+	+	-	+	-	+	-	-	+	-	+
-	-	+	-	+	-	+	+	-	-	+	+	-	+	-

H16-V

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	-	-	-	-	-	-	-	-
+	+	+	-	-	-	-	+	+	+	+	-	-	-	-
+	+	+	-	-	-	-	-	-	-	-	+	+	+	+
+	-	-	+	+	-	-	+	+	-	-	+	+	-	-
+	-	-	+	+	-	-	-	-	+	+	-	-	+	+
+	-	-	-	-	+	+	+	-	+	-	+	-	+	-
+	-	-	-	-	+	+	-	+	-	+	-	+	-	+
-	+	-	+	-	+	-	+	+	-	-	-	-	+	+
-	+	-	+	-	+	-	-	-	+	+	+	+	-	-
-	+	-	-	+	-	+	+	-	-	+	-	+	+	-
-	+	-	-	+	-	+	-	+	+	-	+	-	-	+
-	-	+	+	-	-	+	+	-	+	-	-	+	-	+
-	-	+	+	-	-	+	-	+	-	+	+	-	+	-
-	-	+	-	+	+	-	+	-	-	+	+	-	-	+
-	-	+	-	+	+	-	-	+	+	-	-	+	+	-

[18]Discussed the different projections of H16-I - H16-V top=3, 4 and 5 dimensions by observing the repeat and mirror image patterns of the runs of these projected designs. [19]Gave a complete collection of the non-isomorphic projections to p = 2, 3,….., 14 dimensions for each of these five designs. Table 2 gives the number of non-isomorphic designs obtained from each of the five matrices when the design is projected onto p = 2, 3, 4, 5 and 6 dimensions.

Table 2: Number of non-isomorphic projections of H16-I - H16-V

p	H16-I	H16-II	H16-III	H16-IV	H16-V	Total
2	1	1	1	1	1	1
3	2	3	3	3	3	3
4	3	5	5	5	5	5
5	4	10	11	10	10	11
6	5	18	26	18	20	27

Since a sub-matrix of one Hadamard matrix may be isomorphic to a sub-matrix of another Hadamard matrix, the total number of distinct N x p sub-matrices from HI6-I - HI6-V is less than the sum of numbers in each row. The last column in the above Table 2 gives the total number of non-isomorphic projections based on the five matrices HI6-I - HI6-V.

Combined Array Results for the 16-Run Non-Equivalent Designs

Following the steps in the algorithm, we now discuss the combined array concept for the 16-run non-equivalent designs:

There are 15 independent columns for studying factor effects and16 design points in all the five 16-run non-equivalent designs.

Case I: p=3

Suppose we have three factors then we need to choose three columns from a given design. For p = 3, there are three non-isomorphic designs from all the five 16-run designs, *viz*. design 3.1, 3.2 and 3.3. Now there are two possibilities *viz*. one can allocate two columns to the control factors an done to the noise factor or one to the control factor and two to the noisefactors. Consider the first possibility:

r = 2, s = 1

Allocate two columns to the control factors and one to the noisefactor, three possibilities are:

Table 3:Possible Allocations for r = 2, s = 1

D.No.	C	N	CxN	CxC
1	1,2	3	13, 23	12
2	1,3	2	12, 32	13
3	2,3	1	21, 31	23

where column C indicates the allocation of control factors in the indicated column of Table 3, column N indicates the allocation of noise factors in the indicated column, column CxN indicates the interactions between control and noise factors and column CxC indicates the interactions between the control factors. The supporting model would be:

$$y = \beta_1 x_1 + \beta_2 x_2 + \gamma_1 z_1 + \delta_{11} x_1 z_1 + \delta_{21} x_2 z_1 + \beta_{12} x_1 x_2 + \varepsilon \qquad (6.1)$$

There are six parameters to be estimated including the C x C interactions. For design 3.1, D-efficiency is zero. Designs 3.2 and 3.3estimate all the six parameters of the above model in eight runs only(after deleting the repeats). The entire information is given in Table4. In the Table after giving the design number we indicate the Hadamard matrix from which it is obtained, we then give the column allocation of the selected design in parenthesis and the number in the second parenthesis gives the number of distinct runs in the design. We give only one allocation as the other allocations can be obtained by relabeling the control and noise factors of the given allocation.

Table 4: Design 3.2, H16-I, (1,2,4), (8)

D. No.	C	N	CxN	CxC	D	D_s
1	1,2	4	14, 24	12	1	1, 1, 1, 1, 1,1

Design 3.3, H16-II, (4,8,12), (8)

D. No.	C	N	CxN	CxC	D	D_s
1	4,8	12	412, 812	48	1	1, 1, 1, 1, 1,1

We obtain only one non-isomorphic interaction graph for both the designs. In the graph each node represents a main effect and eachline represents a two-factor interaction connecting the two nodes. Sincea graph can represent different allocations of control and noise factors, we do not put the labels for the nodes. The D value of each design isgiven at the bottom of each graph.

(a) r = 1 and s = 2

The number of parameters reduce to five and D value for design 3.1 = 0. For designs 3.2 and 3.3,D value = 1 and Ds values for all the effects in all the three allocations come out to be 1. We obtain only one non-isomorphic interaction graph from both the designs.

Case II: p = 4

When we have four factors then we need to choose four columns from a given design. There are five non-isomorphic designs from all the five Hadamard matrices viz. design 4.1, 4.2, 4.3, 4.4 and 4.5. There are three different possibilities in which four factors can be divided into control and noisefactors:

(a) r = 3, s = 1 (b)r=1,s=3 (c)r=2,s=2

Consider the first possibility:

(a) r = 3, s = 1

Allocate three columns to the control factors and one to the noise factor. If one does not includethe C x C interactions, then there are inall seven parameters to be estimated (equivalent to the case(b) r = 1, s = 3). The corresponding model excluding $\beta_0$would be:

$$y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \gamma_1 z_1 + \delta_{11} x_1 z_1 + \delta_{21} x_2 z_1 + \delta_{31} x_3 z_1 + \varepsilon \quad (6.2)$$

Designs 4.1 and 4.2 consist of 8 distinct runs and therefore enable us to estimate all the seven parameters in 8 runs only. Design 4.2 performs better than design 4.1 (though it has the same D-efficiency) as it provides more flexibility in the allocation of control and noise factors. In the class of designs having equal efficiency, we call a design to be the best if it provides maximum flexibility in the allocation of control and noisefactors. We thus give below results for design 4.2 only. Also, we give only one allocation as the other allocations can be obtained by renaming the control and noise factors of the given allocation.

Table 5:Design 4.2, H16-II, (1,2,4,7), (8)

D. No.	C	N	CxN	CxC	D	D_s
1	1,2,4	7	17, 27, 47	-	1	1, 1, 1, 1, 1,1, 1

We obtain only one non-isomorphic interaction graph.

It may be noted here that this design performs better than the12 x 4 sub-matrix discussed by [20]and the three 20 x 4 sub-matrices discussed by [21]as it estimates all the 7 parameters in lesser number of runs and with higher D-efficiency.

If one includes the set of C x C interactions, then there are ten parameters to be estimated: The corresponding model excluding $\beta_0$would be:

$$y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \gamma_1 z_1 + \delta_{11} x_1 z_1 + \delta_{21} x_2 z_1 + \delta_{31} x_3 z_1 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 + \epsilon \quad (6.3)$$

For designs 4.1 and 4.2, D value = 0. Design 4.3 consists of all the 16runs of the regular 16-run design and therefore enables us to estimate all the parameters with high efficiency. However, this design uses 16 runs to estimate only ten parameters. We therefore search for a design which estimates the required number of parameters in minimum number of runs. Designs 4.4 and 4.5 consist of 12 distinct design points (after deleting the repeats) and estimate all the ten parameters with equal efficiency. As design 4.4 provides more flexibility in the allocation of control and noise factors we give below results only for this design. We give below only one allocation as the other allocations can be obtained by renaming the control and noise factors of the given allocation:

Table 6: Design 4.4, HI6-II, (1,4,8,12), (12)

D. No.	C	N	CxN	CxC	D	D_s
1	1,4,8	12	112, 412, 812	14, 18, 48	.85	1, .67, .67, .67, .67, .67, .67, .67, .67, .67

We obtain only one non-isomorphic interaction graph.

It is pertinent to mention here that the 12 x 4 sub-matrix discussed by [20]performs better than the three non-equivalent 20 x 4 sub-matrices discussed by [21]and also the above design as it estimates all the 10 parameters in minimum number of runs, though with lower D-efficiency,

(c) r=2, s =2

Allocate two columns to the control factors and two to the noisefactors. There are in all 9 parameters to be estimated (including C x C interactions). The corresponding model excluding $\beta_0$would be:

$$y = \beta_1 x_1 + \beta_2 x_2 + \gamma_1 z_1 + \gamma_2 z_2 + \delta_{11} x_1 z_1 + \delta_{21} x_2 z_1 + \delta_{12} x_1 z_2 + \delta_{22} x_2 z_2 + \beta_{12} x_1 x_2 + \epsilon \tag{6.4}$$

For designs 4.1 and 4.2, D value = 0. Design 4.3 estimates the required number of parameters with D-efficiency = 1. However, we do not prefer to use this design as this design uses 16 runs for estimating only nine parameters. Designs 4.4 and 4.5, both consist of 12 design points(after deleting the repeats) and estimate the required number of parameters with equal efficiency. However, Design 4.4 performs better as it provides more flexibility in the allocation of control and noise factors. We give below results for this design:

Table 7:Design 4.4, H16-II, (1,4,8,12), (12)

D. No.	C	N	CxN	CxC	D	D_s
1	1,4	8, 12	18,112, 48, 412	14	.88	1, .89, .67, .67, .67, .67, .67, .67, .89

The other allocations can be obtained by renaming the control and noise factors o fthe given allocation. We obtain only one non-isomorphic interaction graph. It may be mentioned here that the 20 x 4 sub-matrix estimates the parameters of the above linear model with the same D-efficiency as discussed by [21]. However, in this case the 12 x 4 sub-matrix discussed by Kaul and Chowdhury [20]performs better than both these designs as it requires lesser number of experimental runs.

Case III: p = 5

When we have five factors then we need to choose five columns from a given design. For p = 5, there are eleven non-isomorphic designs from all the five Hadamard matrices *viz*. design 5.1, 5.2, …, and 5.11. Now there are four different possibilities in which five factors can be divided into control and noise factors:

(a) r = 4, s =1, (b) r = 3, s = 2, (c)r = 1, s = 4, (d) r = 2, s = 3

Consider the first possibility:

(a) r =4,s= 1

Allocate four columns to the control factors and one to the noisefactor. In all there are 15 parameters to be estimated including the Cx C interactions. For designs 5.1, 5.2, 5.3, 5.5, 5.7, 5.9, 5.10 and 5.11, D value = 0. Among three designs having non-zero D-efficiency, design 5.6 performs the best. This design contains all the 16 runs of the regular 2^{5-1}fractional factorial design with resolution V and thus allows all the 15 parameters to be estimated with high efficiency. We give below results for this design:

Table 8:Design 5.6, H16-II, (1,4,6,8,11), (16)

D. No.	C	N	CxN	CxC	D	D_s
1	1,4,6,8	11	111, 411, 611, 811	14, 16, 18, 46, 48, 68	1	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1

The other allocations can be obtained by renaming the Control and noise factors of the given allocation. We obtain only one non-isomorphic interaction graph.It may be mentioned here that this design estimates all the 15parameters of the above linear model with maximum D-efficiency in minimum number of runs in comparison to the nine 20 x 5 sub-matrices discussed by [21].

(b) r = 3, s = 2

Allocate three columns to the control factors and two to the noise factors. There are in all 14 parameters to be estimated (including the C x C interactions). For designs 5.1, 5.2, 5.3, 5.5. 5.9 and 5.11, D-efficiency is = 0. Among other designs having non-zero D-efficiency, again design 5.6 performs the best:

Table 9:Design 5.6, H16-II, (1,4,6,8,11), (16)

D. No.	C	N	CxN	CxC	D	D_s
1	1,4,6	8, 11	18,111, 48, 411, 68, 611	14, 16, 46	1	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1

The other allocations can be obtained by renaming the control and noise factors of the given allocation. We obtain only one non-isomorphic interaction graph.

It may be mentioned here that this design estimates all the 14 parameters of the above linear model with maximum D-efficiency as compared to the nine 20 x 5 sub-matrices discussed by[20].

(c) r = 1, s = 4

Allocate one column to the control factor and four to the noise factors. Only 9 parameters are to be estimated in this case (equivalent to the case r = 4, s = 1 when no C x C interactions are included). D value is 0 for design 5.1. Designs 5.2, 5.3, 5.4, 5.6 and 5.7 estimate the required number of parameters with D-efficiency = 1. However, they use 16 runs to estimate only nine parameters. Designs 5.5 and 5.9 estimates the required number of parameters with equal D-efficiency in 12 runs only (after deleting the repeats). As design 5.9 provides more flexibility in the allocation of control and noise factors we give below results for this design:

Table 10: Design 5.9, HI6-II, (4,5,8,9,12), (12)

D. No.	C	N	CxN	CxC	D	D_s
1	4	5, 8, 9, 12	45, 48, 49, 412	-	.88	.89, .89, .67, .67, .67, 1, .67, .67, .67

The other allocations can be obtained by renaming the control and noise factors of the given allocation. We obtain only one non-isomorphic interaction graph.

It may be noted here that this design estimates all the 9 parameters of the above linear model with maximum D-efficiency in comparison to the two 12 x 5 sub-matrices as discussed by [19]and the nine 20 x 5sub-matricesdiscussed by[21].

(d) r = 2, s = 3

Allocate two columns to the control factors and three to the noise factors. In this case there are 12 parameters to be estimated including theCxC interactions. For designs 5.1, 5.5 and 5.9, D-efficiency is = 0.Among other designs having non-zero D-efficiency, only design 5.11 estimate*s* all the 12 parameters of the above model in 14 runs (though there exist designs with higher D-efficiency but they use more runs). We give below the result for this design:

Table 11: Design 5.11, HI6-III, (2,4,8,10,12), (14)

D. No.	C	N	CxN	CxC	D	D_s
1	8,12	2, 4, 10	82, 84, 810, 122, 124, 1210	812	.72	.38, .38, .43, .43, .29, .29, .57, .61, .57, .29, .61, .36

We obtain only one non-isomorphic interaction graph.

It may be mentioned here that of the nine 20 x 5 non-equivalent sub-matrices discussed by [21],one of them estimates all the 12 parameters of the above linear model with the same D-efficiency using the same number of experimental runs.

Case IV: p = 6

For p = 6, there are 27 non-isomorphic designs from all the five Hadamard matrices viz. design 6.1, 6.2, …, and 6.27. Now there are five different possibilities in which six factors can be divided into control and noise factors:

(a) r = 5, s = 1 (b) r = 4, s = 2(c) r = 1, s = 5(d) r = 2, s = 4(e) r = 3,s = 3

Consider the first possibility:

(a) r = 5, s = 1

Allocate five columns to the control factors and one to the noisefactor. There are 11 parameters to be estimated excluding the C x C interactions. Now one can include four C x C interactions, which can be chosen in$\binom{10}{4}$ ways. In all there will be 1260 (210 x 6) cases. Out of27 designs, D value = 0 for designs 6.1, 6.2, 6.3, 6.5, 6.10, 6.11, 6.12,6.16, 6.17, 6.21, 6.22, 6.25, 6.26, and 6.27. Out of other deigns having on-zero D-efficiency, design 6.4 estimates 15 parameters with maximum D-efficiency. The following allocations of control and noise factors have come out to be the best for this design:

Table 12: Design 6.4, H16-I, (1,2,3,4,8,13), (16)

D. No.	C	N	CxN	CxC	D	D_s
1	1, 2, 3, 4, 8	13	113, 213, 313, 413, 813	24, 28, 34, 38	1	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1
2	1, 2, 3, 4, 13	8	18, 28, 38, 48, 138	24, 213, 34, 313	1	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1
3	1, 2, 3, 8, 13	4	14, 24, 34, 84, 134	28, 213, 38, 313	1	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1

We obtain only one non-isomorphic interaction graph.

Next, from design 6.13 we get another non-isomorphic interaction graph. The allocations of control and noise factors that have come out tobe best for this design are:

Table 13: Design 6.13, H16-II, (1,4,6,8,11,12), (16)

D. No.	C	N	CxN	CxC	D	D_s
1	4, 6, 8, 11, 12	1	41, 61, 81, 111, 121	46, 48, 411, 811	.91	1, 1, 1, 1, .5, 1, 1, 1, 1, 1, .5, 1, .5, .5, 1
2	4, 6, 8, 11, 12	1	41, 61, 81, 111, 121	46, 48, 68, 811	.91	1, 1, 1, 1, .5, 1, 1, 1, 1, 1, .5, 1, .5, .5, 1
3	4, 6, 8, 11, 12	1	41, 61, 81, 111, 121	46, 411, 611, 811	.91	1, 1, 1, 1, .5, 1, 1, 1, 1, 1, .5, 1, .5, .5, 1
4	4, 6, 8, 11, 12	1	41, 61, 81, 111, 121	46, 68, 611, 811	.91	1, 1, 1, 1, .5, 1, 1, 1, 1, 1, .5, 1, .5, .5, 1

Out of seven designs, viz. 6.6, 6.7, 6.8, 6.9, 6.14, 6.19, and 6.24, all having same D-efficiency, we get three more non-isomorphic interaction graphs. We give here the allocations of control and noise factors that have come out to best for the design which provides maximum flexibility in the allocation of control and noise factors:

Table 14:Design 6.14, H16-II, (4,5,6,7,8,12), (16)

D. No.	C	N	CxN	CxC	D	D_s
1	4, 5, 6, 8, 12	7	47, 57, 67, 87, 127	48, 412, 58, 512	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
2	4, 5, 6, 8, 12	7	47, 57, 67, 87, 127	48, 412, 58, 612	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
3	4, 5, 6, 8, 12	7	47, 57, 67, 87, 127	48, 412, 512, 68	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
4	4, 5, 6, 8, 12	7	47, 57, 67, 87, 127	48, 412, 68, 612	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
5	4, 5, 6, 8, 12	7	47, 57, 67, 87, 127	48, 58, 512, 612	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
6	4, 5, 6, 8, 12	7	47, 57, 67, 87, 127	48, 512, 68, 612	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
7	4, 5, 6, 8, 12	7	47, 57, 67, 87, 127	412, 58, 512, 68	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
8	4, 5, 6, 8, 12	7	47, 57, 67, 87, 127	412, 58, 68, 612	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
9	4, 5, 6, 8, 12	7	47, 57, 67, 87, 127	58, 512, 68, 612	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
10	4, 5, 7, 8, 12	6	46, 56, 76, 86, 126	48, 412, 58, 512	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
11	4, 5, 7, 8, 12	6	46, 56, 76, 86, 126	48, 412, 58, 712	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
12	4, 5, 7, 8, 12	6	46, 56, 76, 86, 126	48, 412, 512, 78	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
13	4, 5, 7, 8, 12	6	46, 56, 76, 86, 126	48, 412, 78, 712	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
14	4, 5, 7, 8, 12	6	46, 56, 76, 86, 126	48, 58, 512, 712	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
15	4, 5, 7, 8, 12	6	46, 56, 76, 86, 126	48, 512, 78, 712	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
16	4, 5, 7, 8, 12	6	46, 56, 76, 86, 126	412, 58, 512, 78	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
17	4, 5, 7, 8, 12	6	46, 56, 76, 86, 126	412, 58, 78, 712	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
18	4, 5, 7, 8, 12	6	46, 56, 76, 86, 126	58, 512, 78, 712	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
19	4, 6, 7, 8, 12	5	45, 65, 75, 85, 125	48, 412, 68, 612	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
20	4, 6, 7, 8, 12	5	45, 65, 75, 85, 125	48, 412, 68, 712	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
21	4, 6, 7, 8, 12	5	45, 65, 75, 85, 125	48, 412, 612, 78	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
22	4, 6, 7, 8, 12	5	45, 65, 75, 85, 125	48, 412, 78, 712	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
23	4, 6, 7, 8, 12	5	45, 65, 75, 85, 125	48, 68, 612, 712	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
24	4, 6, 7, 8, 12	5	45, 65, 75, 85, 125	48, 612, 78, 712	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
25	4, 6, 7, 8, 12	5	45, 65, 75, 85, 125	412, 68, 612, 78	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
26	4, 6, 7, 8, 12	5	45, 65, 75, 85, 125	412, 68, 78, 712	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
27	4, 6, 7, 8, 12	5	45, 65, 75, 85, 125	68, 612, 78, 712	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
28	5, 6, 7, 8, 12	4	54, 64, 74, 84, 124	58, 512, 68, 612	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
29	5, 6, 7, 8, 12	4	54, 64, 74, 84, 124	58, 512, 68, 712	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
30	5, 6, 7, 8, 12	4	54, 64, 74, 84, 124	58, 512, 612, 78	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
31	5, 6, 7, 8, 12	4	54, 64, 74, 84, 124	58, 512, 78, 712	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
32	5, 6, 7, 8, 12	4	54, 64, 74, 84, 124	58, 68, 612, 712	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
33	5, 6, 7, 8, 12	4	54, 64, 74, 84, 124	58, 612, 78, 712	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
34	5, 6, 7, 8, 12	4	54, 64, 74, 84, 124	512, 68, 612, 78	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
35	5, 6, 7, 8, 12	4	54, 64, 74, 84, 124	512, 68, 78, 712	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5
36	5, 6, 7, 8, 12	4	54, 64, 74, 84, 124	68, 612, 78, 712	.83	1, 1, 1, .25, .25, 1, 1, 1, 1, .5, .5, .5, .5, .5, .5

Table 15: Design 6.19, H16-III, (1,2,4,8,10,12),(16)

D. No.	C	N	CxN	CxC	D	D_s
1	2, 4, 8, 10, 12	1	21, 41, 81, 101, 121	24, 210, 212, 810	.83	.5, 1, .5, 1, 1, 1, .5, 1, .5, 1, 1,.5, .5, .5, .25
2	2, 4, 8, 10, 12	1	21, 41, 81, 101, 121	24, 210, 212, 101	.83	1, .5, .5, .5, 1, 1, 1, 1, .5, .5, 1, .5, .5, .5, .25
3	2, 4, 8, 10, 12	1	21, 41, 81, 101, 121	24, 48, 410, 810	.83	.5, 1, 1, 1, .5, 1, .5, 1, 1, 1, .5, .5, .5, .5, .25
4	2, 4, 8, 10, 12	1	21, 41, 81, 101, 121	24, 48, 410, 1012	.83	1, .5, 1, 1, .5, 1, 1, .5, 1, 1, .5, .5, .5, .5, .25

Table 16: Design 6.24, H16-III, (2,4,8,9,10,14), (16)

D. No.	C	N	CxN	CxC	D	Ds
1	2, 4, 8, 10, 14	9	29, 49, 89, 109, 149	28, 48, 810, 814	.83	.5, .5, 1, .5, .5, 1, .67, .67, 1, .67, .67, .67, .67, .67, .67
2	2, 4, 9, 10, 14	8	28, 48, 98, 108, 148	29, 49, 910, 914	.83	.5, .5, 1, .5, .5, 1, .67, .67, 1, .67, .67, .67, .67, .67, .67

(b) r = 4, s = 2

Allocate four columns to the control factors and two to the noise factors. There are 14 parameters to be estimated excluding the C x C interactions. Now one can include one C x C interaction, which can be chosen in 6 ways. In all there will be 90 (15 x 6) cases. Out of 27designs, D value = 0 for 21 designs. Out of other deigns, designs 6.13, 6.15 and 6.19 estimate all the 15 parameters with maximum D-efficiency. However, designs 6.13 and 6.15 provide more flexibility in the allocation of control and noise factors. The following allocations of control and noise factors have come out to be best for these designs:

Table 17: Design 6.13, H16-II, (1,4,6,8,11,12), (16)

D. No.	C	N	CxN	CxC	D	Ds
1	1, 4, 8, 12	6, 11	16, 111, 46, 411, 86, 811, 126, 1211	48	.76	1, .5, .5, .25, .25, .5, .5, .5, .5, 1, .5, 1, .25, .25, .5
2	1, 4, 11, 12	6, 8	16, 18, 46, 48, 116, 118, 126, 128	411	.76	1, .5, .5, .25, .25, .5, .5, .5, .5, 1, .5, 1, .25, .25, .5
3	1, 6, 8, 12	4, 11	14, 111, 64, 611, 84, 811, 124, 1211	68	.76	1, .5, .5, .25, .25, .5, .5, .5, .5, 1, .5, 1, .25, .25, .5
4	1, 6, 11, 12	4, 8	14, 18, 64, 68, 114, 118, 124, 128	611	.76	1, .5, .5, .25, .25, .5, .5, .5, .5, 1, .5, 1, .25, .25, .5

Design 6.15, H16-II, (4,5,6,8,9,12), (16)

D. No.	C	N	CxN	CxC	D	Ds
1	4, 5, 8, 12	6, 9	46, 49, 56, 59, 86, 89, 126, 129	412	.76	.5, .5, .5, .25, .5, .5, 1, .5, 1,.5, .5,1, .5, .25, .5
2	4, 5, 8, 12	6, 9	46, 49, 56, 59, 86, 89, 126, 129	512	.76	.5, .5, .5, .25, .5, .5, 1, .5, 1, .5, .5, 1, .5, .25, .5
3	4, 5, 9, 12	6, 8	46, 48, 56, 58, 96, 98, 126, 128	412	.76	.5, .5, .5,.25, .5,.5,1, .5,1, .5, .5, 1, .5, .25, .5
4	4, 5, 9, 12	6, 8	46, 48, 56, 58, 96, 98, 126, 128	512	.76	.5, .5, .5, .25, .5, .5, 1, .5, 1, .5, .5, 1, .5, .25, .5

We obtain only non-isomorphic interaction graph.

(c) r = 1, s=5

Allocate one column to the control factor and five to the noise factors. There are in all 11 parameters to be estimated as there are no CxC interactions. Out of27 designs, D value = 0 for only four designs. Out of designs having D value = 1, design 6.5 provides maximum flexibility in the allocation of control and noise factors. The following allocations of control and noise factors have come out to be the best for this design:

Table 18: Design 6.5, H16-I, (1,2,4,7,8,11), (16)

D. No.	C	N	CxN	CxC	D	Ds
1	1	2, 4, 7, 8, 11	12, 14, 17, 18, 111	-	1	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1

The other allocations can be obtained by renaming the control and noise factors of the given allocation. We obtain only one non-isomorphic interaction graph.

It is worth mentioning here that this design performs better than the design discussed by Kaul and Chowdhury (2021,b) as it estimates all the 11 parameters in lesser number of runs and with higher D-efficiency.

(d) r=2, s = 4

Allocate two columns to the control factors and four to the noisefactors. There are 15 parameters to be estimated including the C x C interactions. Out of 27 designs, D value = 0 for 22 designs. Out of other deigns having non-zero D-efficiency, designs 6.19 and 6.24 estimate all the 15 parameters with maximum D-efficiency. The following allocations of control and noise factors have come out to be best for these designs:

Table19: Design 6.19, HI6-III, (1,2,4,8,10,12), (16)

D. No.	C	N	CxN	CxC	D	D_s
1	1,10	2, 4, 8, 12	12, 14, 18, 112, 102, 104, 108, 1012	110	.83	1, 1, .67, .67, .67, .67, .67, .67, .67, .67, .5, .5, .5, .5, 1

Design 6.24, HI6-III, (2,4,8,9,10,14), (16)

D. No.	C	N	CxN	CxC	D	D_s
1	8, 9	2, 4, 10, 14	82, 84, 810, 814, 92, 94, 910, 914	89	.83	1, 1, .5, .5, .5, .5, .67, .67, .67, .67, .67, .67, .67, .67, 1

We obtain only one non-isomorphic interaction graph.
It may be mentioned here that this design performs better than the design given by [20]as it estimates all the 15parameters in lesser number of runs and with higher D-efficiency.

(d) r =3,s=3

Allocate three columns to the control factors and three to the noise factors. There are 15 parameters to be estimated. No C x C interactions can be estimated owing to non-availability of degrees of freedom. Out of 27 designs, D value = 0 for 18 designs. Out of other designs having non-zero D-efficiency, designs 6.3, 6.4 and 6.6 estimate all the 15 parameters with maximum D-efficiency and provide equal flexibility in the allocation of control and noise factors. The following allocations of control and noise factors have come out to be best for these designs:

Table20: Design 6.3, HI6-I, (1,2,3,4,8,12), (16)

D. No.	C	N	CxN	CxC	D	D_s
1	1, 2, 3	4, 8, 12	14, 18, 112, 24, 28, 212, 34, 38, 312	-	1	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1
2	4, 8, 12	1, 2, 3	41, 42, 43, 81, 82, 83, 121, 122, 123	-	1	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1

Design 6.4, HI6-III, (1, 2, 3, 4, 8, 13), (16)

D. No.	C	N	CxN	CxC	D	D_s
1	1, 2, 3	4, 8, 13	14, 18, 113, 24, 28, 213, 34, 38, 313	-	1	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1
2	4, 8, 13	1, 2, 3	41, 42, 43, 81, 82, 83, 131, 132, 133	-	1	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1

Design 6.6, HI6-II, (1,2,3, 4,8,12), (16)

D. No.	C	N	CxN	CxC	D	D_s
1	1, 2, 3	4, 8, 12	14, 18, 112, 24, 28, 212, 34, 38, 312	-	1	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1
2	4, 8, 12	1, 2, 3	41, 42, 43, 81, 82, 83, 121, 122, 123	-	1	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1

We obtain only one non-isomorphic interaction graph.
we now illustrate with the help of an example how combined array approach using a 16-run non-regular design can reduce the number of runs in contrast with the product array approach as well as allows more flexibility in the selection of estimable effects:
An experiment was carried out to study the effect of a number of variables on yield of paddy crop. Two controllable factors *viz*. variety of fertilizer (A), type of seeds (B) and three uncontrollable factors *viz*. temperature (r), humidity (s)and rainfall (t)were identified for the purpose. Also, the interaction (AB) between the two controllable factors was important and the experimenter wanted to estimate it. If we use the product array approach for the purpose, we first construct a 2^2 full factorial design for the control array that can estimate the main effects A, Band the interaction effect AB. We then construct a 2^{3-1} fractional factorial plan for the noise array that estimates the three main effects r, s and t. The resulting 16-run product array allows us to estimate the five main effects A, B, r, s and t, the seven two-factor interactions AB, Ar, As, At, Br, Bs, Bt and 3 higher order interactions. As the 3 higher order interactions are less likely to be important, we can construct a linear model consisting of the five main effects and the seven two-factor interactions. To estimate the 12 parameters of the above linear model, if we allocate the two control factors to columns 8 and 12 and the three noise factors to columns 2, 4, and 10 of H16-III, then it allows us to estimate all the 12 parameters in 14 runs only, with D-efficiency = 0.72.

CONCLUSIONS

Since Taguchi's product arrays are often quite large, thus to reduce the number of runs, in this paper we have generated non-regular combined arrays based on 16-run Hadamard matrices. The non-regular combined designs presented in this paper allow flexible estimation of the effects of interest and also reduce the number of runs. To facilitate the assignment of control factors, noise factors and their interactions in real life problems, we further elaborate this concept by developing non-isomorphic interaction graphs. Some of the interaction graphs are shown in the Annexure.

RECOMMENDATION

In some experiments of agricultural sciences, when we have large number of runs even for a single replication, No researcher/experimenter can afford so many runs for an experiment. The researcher's interest is to identify the important factors in the experiment and so main effects of the factors were of major concern. Combined arrays approach using a 16-run non-equivalent design can reduce the number of runs in contrast with the product array approach as well as allows more flexibility in the selection of estimable effects of interest. So we strongly recommend the combined arrays approach using a 16-run non-equivalent design to study the interaction effects between the controllable factors in experiments of agricultural sciences. For example we can study the effect of a number of variables on any crop yield using the controllable factors *viz*. variety of fertilizers, type of seeds and uncontrollable factors *viz*. temperature, humidity and rainfall.

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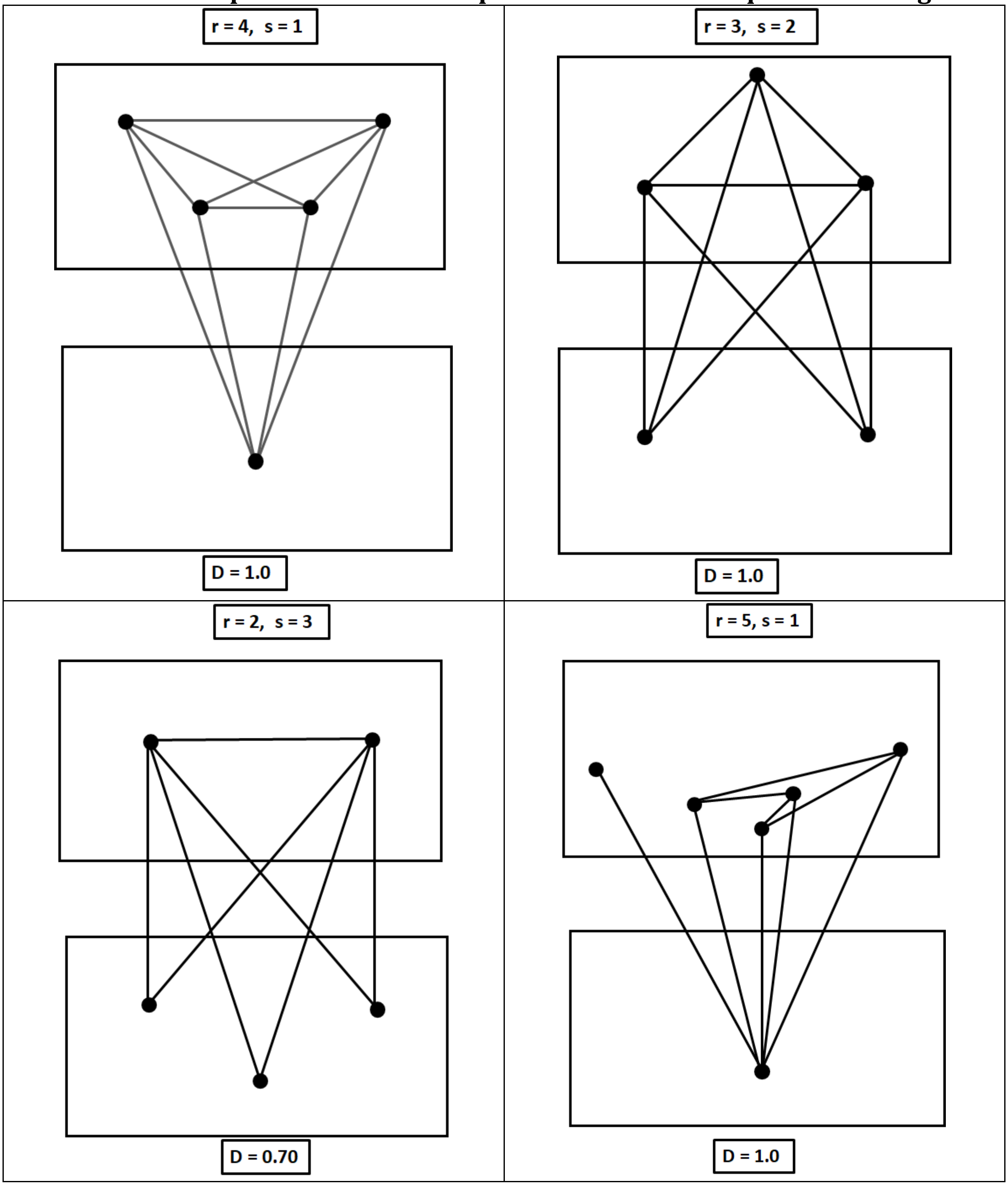
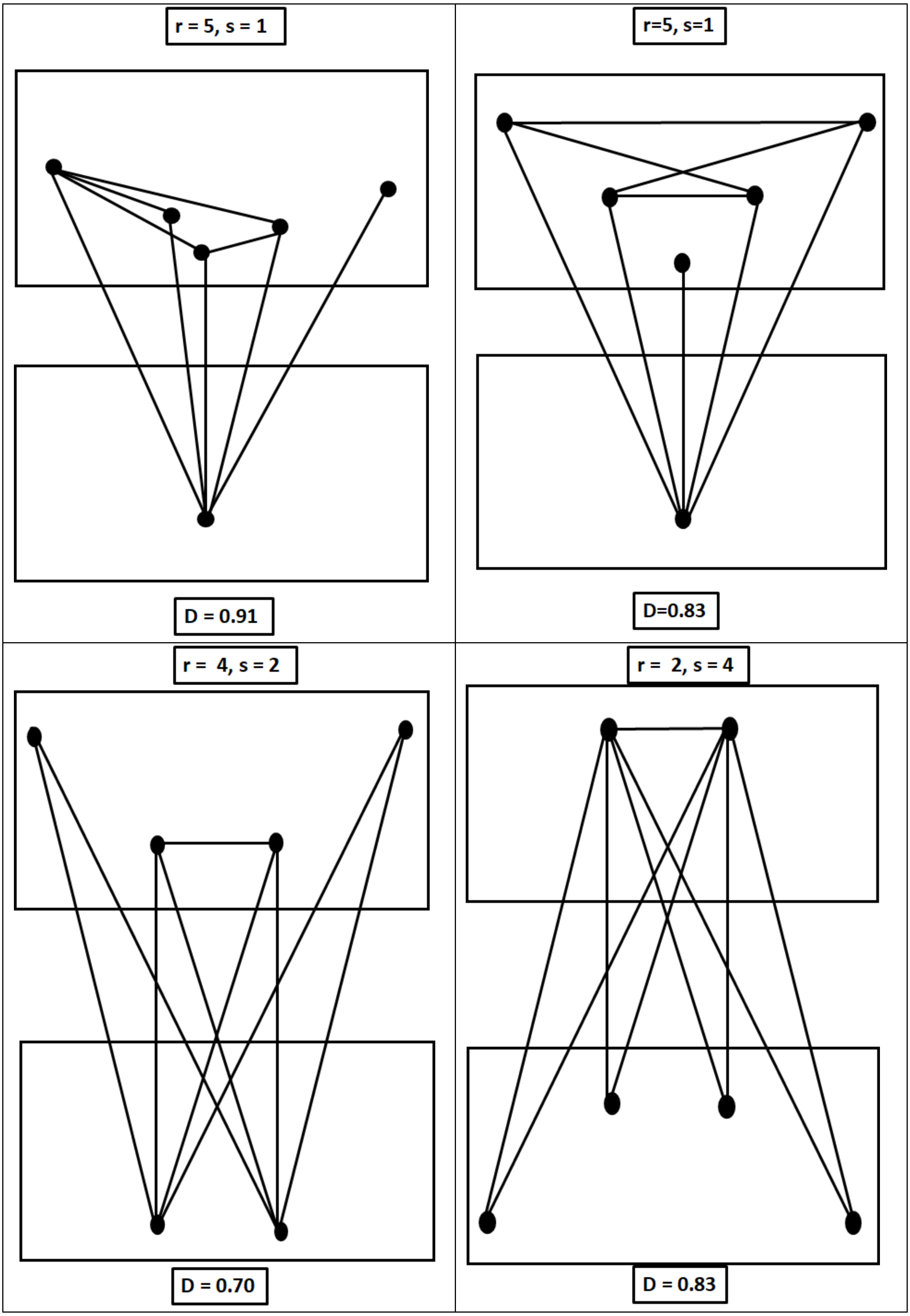
CITATION OF THIS ARTICLE

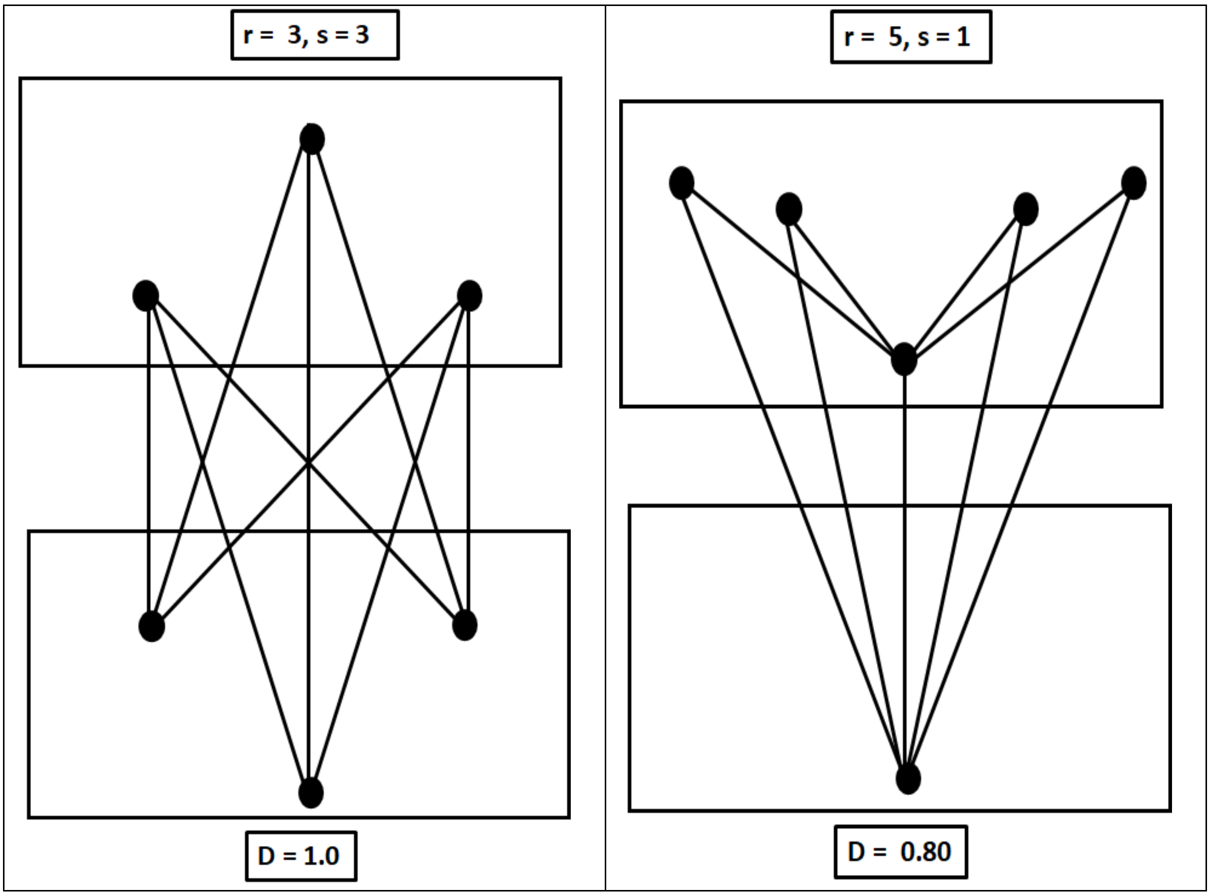
R Kaul, S R Chowdhury and A Bansal. Combined Array Approach for 16-Run Non-equivalent Designs. Bull. Env.Pharmacol. Life Sci., Vol 11[2] January 2022 : 33-46.

ANNEXURE

Non-Isomorphic Interaction Graphs for 16 Run Non-Equivalent Designs




r = 5, s = 1
D = 0.91
r=5, s=1
D=0.83
r = 4, s = 2
D = 0.70
r = 2, s = 4
D = 0.83


r = 3, s = 3
D = 1.0
r = 5, s = 1
D = 0.80