



Using Reproduction Number to Model The Covid-19 Pandemic

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ABSTRACT

Logistic curve which is used to model population growth is used to model the growth of covid -19 cases in India, an important parameter of the growth model is the growth rate r . In the present study the reproduction number of the disease is used to calculate the growth rate r of the covid -19 cases in India and growth curves are worked out for different values of the reproduction number. The duration of the pandemic and inflection point are also estimated for the different scenarios. The observed cases in India are then modeled using the logistic curve and using the reproduction number the duration of pandemic in India and the inflection point is estimated.

Keywords: Reproduction number, Growth Curve, Logistic

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INTRODUCTION

Covid 19 has an unprecedented effect on the human consciousness for the past 7 months and it has transformed the world in variety of ways. The world health organization (WHO) declared this a global pandemic on March 11, 2020[1]. More and more countries are struggling to find out the exact strategies to tackle this pandemic. As of July, 12, 2020 the total confirmed cases are 13042340 with 571689 mortalities worldwide and in India 879487 cases with 23174 mortalities[2]. This all started in Wuhan, China in the month of December, 2019 and then engulfed the whole world very rapidly. Italy, Spain, France, Germany, UK, USA and now Brazil and India are severely affected by this pandemic. More than one dozen candidate vaccines are in different phases of trials, in a desperate search for ways to end this pandemic. But before this can be accomplished the human race is being left with measures like lockdowns, social distancing and use of masks to avoid the pandemic.

With no easy solutions in sight people are concerned with the possible duration of the pandemic, when it will peak and from when the situation will ease for the normal life to return. Various models have been used to find answers to these questions. Different countries are applying different strategies to evade the pandemic. Some are going for lockdowns, partial or complete or intermittent lockdowns while others have adopted the strategy to gain herd immunity to tackle this. Under different scenarios the peaks and duration of the pandemic will of course will be different. In different scenarios the reproduction number of the virus named as R differs and that will affect the peaks and the duration of the pandemic.

The reproduction number of the virus or R is often used to indicate the severity of the infection rate and it estimates the number of people one patient can infect. This reproduction number varies under different strategies of lockdowns like complete, partial or intermittent lockdowns[3][4]. It also varies under different scenarios like social distancing, masking etc. In this study the logistic growth curve is used to model the peaks and duration of the covid 19 in different scenarios like that of lockdown and without lockdown but with social distancing and masks by utilizing the reproduction number of covid 19 in these scenarios. The logistic curve is a growth curve and suits the growth of number of individuals affected by the virus which at the initial stage grows exponentially, and then as immunity develops its growth shows a linear trend and at later stage the growth of the virus slows down. This logic was used in applying logistic curve to find the peaks and duration of the pandemic under different scenario with respect to India. The important parameter of the logistic growth curve ' r ' was replaced by the reproduction number R to generate data for different scenarios

MATERIAL AND METHODS

The number of infected cases started in India on Feb 15 and the lockdown was imposed on 23rd March which continued up to 30th June after which the unlock phase started. During the lockdown also a massive movement of migrants was observed in different parts of India, so the lockdown was not a complete success. So the R (reproduction number) of the disease varied under these phases. This R was used to model the data using the logistic curve. Logistic equation is a common model of population growth, originally given by Pierre-Francois Verhulst in 1838 to describe the self-limiting growth of biological population, this model is formalized by the differential equation:

$$\frac{dP}{dt} = rP \left(1 - \frac{P}{K}\right) \quad \text{eq -1}$$

P = Population Size

t = Time

r = defines the growth rate

K = Carrying capacity

In the equation the term the unrestricted growth of the population is represented by the term rP in the equation. The value of the rate represents the proportional increase in the population P in one unit time. But in unrestricted growth there is exponential growth, where growth rate stays the same regardless of population size and the population grows faster and faster as it gets larger. However in nature, populations generally grow exponentially for a limited time and the growth rate is restricted by the resource availability. In logistic growth, a population's per capita growth rate gets smaller and smaller as population size approaches a maximum imposed by limited resources in the environment, known as the carrying capacity (K). Exponential growth produces a J-shaped curve, while logistic growth produces an S-shaped curve. The number of cases of covid 19 can be modeled by logistic curve as the rate cannot remain constant throughout and the rates reduces as the immunity in the community develops.

The solution to the equation one (with P_0 being the initial population) is

$$P(T) = \frac{K}{1 + \left(\frac{K-P_0}{P_0}\right) e^{-rt}}$$

Where $\lim_{t \rightarrow \infty} P(t) = K$

Which is to say that K is the limiting value of P: the highest value that the population can reach given infinite time (or come close to reaching in finite time).

So in this study r has been replaced by the reproduction number R of the covid 19. The carrying capacity in this case will be the maximum population which will be infected by the disease. The herd immunity in a population is achieved when a certain proportion of the population get infected with the estimates of this proportion varies for different studies. In this study the proportion of 60 % was assumed to be necessary for attaining the herd immunity. Therefore the carrying capacity of the population was taken as sixty percent of the total population of India which means that if 60% of the population gets infected the pandemic will stop. These initial parameters were used to find the duration of the pandemic under different reproduction numbers representing different scenarios and growth curves for different scenario were simulated to predict the duration of the pandemic. In India the R at the first week from march 2-8, 2020 was 3.2. it remained about 2 units for three weeks, from march 9-29, 2020. After march 2020, it started declining and reached around 1.3 in the following week suggesting a stabilization of the transmissibility rate.

As India is a vast country same reproduction number cannot be applied for the whole population but assuming uniformity of the reproduction number for the whole population because of the lack of data, different reproduction number under different scenario were applied for the whole population. The reproduction number is used to measure the transmission potential of a communicable disease, that is the number of individuals infected by a particular infected person. If R_0 is > 1 , the number of infected individuals increases exponentially with the increase of the R_0 value and if R_0 is < 1 the disease is expected to die out in the population. The WHO [5] declared in the statement regarding the SARS-COV-2, dated 23th January, 2020 that the R_0 ranges between 1.4 and 2.5. Liu et al. compared 12 studies and found that R_0 lies in the range between 1.5 and 6.68. This is the number of individuals infected by an infected person in about five days, therefore the growth rate r was calculated for one day by dividing the reproduction number by five.

RESULT

The logistic curve was simulated for different growth rate obtained by the Reproduction number, it was assumed that if sixty percent of the population get infected then herd immunity will eventually stop the pandemic. At present the population of India is 1,380,004,385, sixty percent of which is 828,002,631, which was taken as the carrying capacity K . The results obtained are given in the table 1. For growth rate (r) 0.1, reproduction number 0.5 that is if every infected person infects 0.5 individuals in 5 days, then the pandemic will continue with an increasing rate upto 198th day (Inflection Point) at which almost half of the carrying capacity of the population will get infected and later on it will increase but with a decreasing rate reaching the maximum after 434 days. Similarly for $r=0.15$, $R=0.75$, the inflection point is 132 days and maximum at 290 days, for $r=0.2$, $R=1$, the inflection point is 99 days and maximum at 217 days, for $r=0.25$, $R=1.25$, the inflection point is 79 days and maximum at 174 days, for $r=0.3$, $R=1.5$, the inflection point is 66 days and maximum at 145 days, for $r=0.35$, $R=1.75$, the inflection point is 56 days and maximum at 124 days, for $r=0.4$, $R=2$, the inflection point is 50 days and maximum at 109 days, for $r=0.6$, $R=3$, the inflection point is 33 days and maximum at 73 days for $r=0.8$, $R=4$, the inflection point is 25 days and maximum at 55 days, for $r=1$, $R=5$, the inflection point is 20 days and maximum at 44 days and for $r=1.2$, $R=6$, the inflection point is 17 days and maximum at 37 days. The curves for different scenarios are given in fig: 1

Table : 1 : Inflection Point and final point of the pandemic in India for different growth rate and reproduction number

Growth rate r	Reproduction No. R_0	Inflection Point (Days)	Final (Days)
0.1	0.5	198	434
0.15	0.75	132	290
0.2	1	99	217
0.25	1.25	79	174
0.3	1.5	66	145
0.35	1.75	56	124
0.4	2	50	109
0.6	3	33	73
0.8	4	25	55
1.0	5	20	44
1.2	6	17	37
0.09	.45	221	474

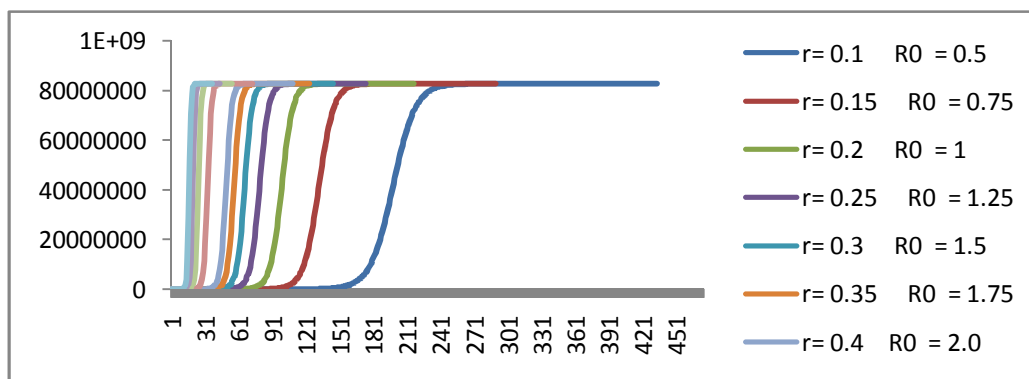


Fig. 1: Logistic curves for different values of r and R_0

In order to fit the observed values i.e. the number of observed cases in India different values of the r growth rate were tried and ultimately for $r = 0.09$ and $R = 0.45$ we observed the fit given in fig - 2.

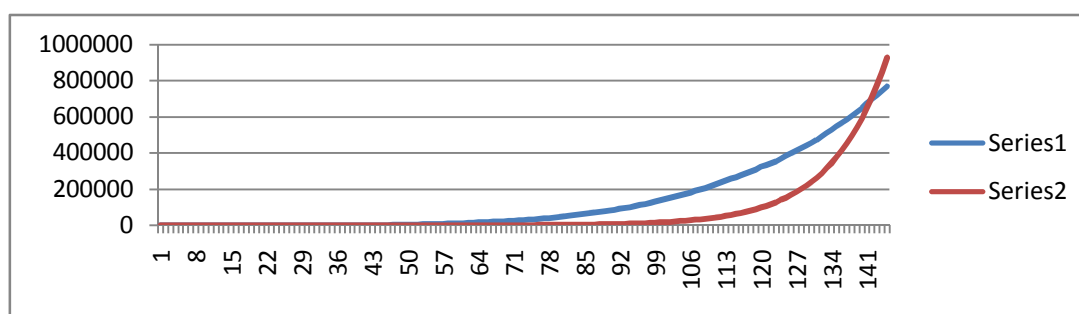


Fig. 2 : The observed and fitted curves for incidences in India

So the present trend has an r nearby the value of 0.09 which means that $R = 0.45$ which means up to 10th July, 2020, an infected person was infecting on an average 0.45 person in 5 days and if this trend continues the peak of the pandemic will be reached after 221 days and after 474 days it will end in India

Fig.3

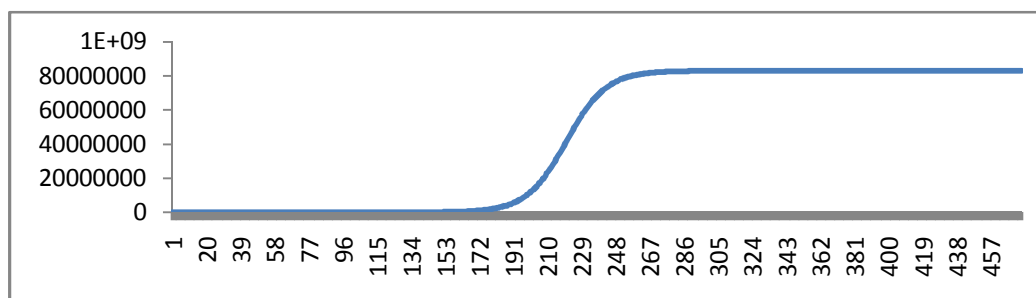


Fig: 3 The growth curve of disease incidences in India

CONCLUSION

Logistic curve which is used to model population growth is utilized to model the growth of covid -19 cases in India. Logistic growth curve initially increases with an increased rate and as the number of infected cases increase the rate of growth decreases which can be explained by increase in herd immunity. Therefore this curve is used for working out the evolution of number of cases of pandemic in India. An important parameter of the growth model is the growth rate r . In the present study the reproduction number of the disease is used to calculate the growth rate r of the covid -19 cases in India and growth curves are worked out for different values of the reproduction number. India is a big country and one estimate of the reproduction number is generalization but nevertheless it gives us an idea of how the disease can progress in the uncertain situation prevailing today. The duration of the pandemic and inflection point are also estimated for the different scenarios. The logistic curve is then used to model observed cases of Covid-19 in India and then using the reproduction number so obtained the duration of pandemic in India and the inflection point is estimated.

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