



ORIGINAL ARTICLE

Sliding Mode Control and Stability Analysis of a DC-DC Converter Buck

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ABSTRACT

In this paper, Sliding Mode Control (SMC) for Buck, converters are proposed, tested and compared. Also a detailed analysis is undertaken to explore the stability and bifurcation pattern of the nonlinear phenomena in the Buck DC-DC converter leading to a better understanding of its dynamics. First a nonlinear system modelling is derived for open-loop Buck converter. This model is then extended for the closed-loop system implementing a proportional - integral (PI) compensation scheme. After the initial analysis of this converter and stability region identification, we utilize the MATLAB to analyze the detailed bifurcation scenario as the parameters are varied. The simulation was performed to achieve satisfactory dynamic performance. Finally, we propose a sliding mode controller for the proposed converter.

Keywords: DC-DC converters, chaotic systems, Sliding Mode Control, stability

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INTRODUCTION

Switched mode DC-DC converters are electronic circuits which convert a voltage from one level to a higher or lower one. They are considered to be the most advantageous supply tools for feeding some electronic systems in comparison with linear power supplies which are simple and have a low cost. However, they are inefficient as they convert the dropped voltage into heat dissipation. The switched-mode DC-DC converters are more and more used in some electronic devices such as DC-drive systems, electric traction, electric vehicles, machine tools, distributed power supply systems and embedded systems to extend battery life by minimizing power consumption (Rashid, 2001). One of the requirements for the next generation of power supplies for distributed power systems (DPS) is to achieve high power density with high efficiency. In the traditional front-end converter based on the two-stage approach for high-power three phase DPS, the DC-link voltage coming from the power factor correction (PFC) stage penalizes the second-stage DC-DC converter [1]. This DC-DC converter not only has to meet the characteristics demanded by the load, but also must process energy with high efficiency, high reliability, high power density and low cost [2]. In its simplest terms, the operation of a DC-DC converter can be described as an orderly repetition of a fixed sequence of circuit topologies. The conversion function of the converter is determined by the constituent topologies and the order in which they are repeated. Such toggling between circuit topologies is achieved by placing switches at suitable positions and turning them on and off in such a way that the required topological sequence is produced [3]. Clearly, the absence of a fixed circuit configuration poses a serious problem to the analysis and modelling of DC-DC converters. The major difficulty lies in the fact that the manner in which the system operates is highly nonlinear [4]. In most of the above investigations, sampled-data models or maps of the converters have been derived, and the bifurcation structures have been investigated with the discrete models. Bifurcation denotes for a change in the number of candidate operating conditions of a nonlinear system when a parameter is quasi-statically varied [5]. The candidate operating condition is also an equilibrium point, a periodic solution, or other invariant subset of its limit set, without regard to its stability properties. The parameter being varied is referred to as the bifurcation parameter. A nonlinear dynamical system can exhibit many different kinds of bifurcations as one or more parameters are varied [6].

To improve the performances of the Buck DC-DC converter, a nonlinear control strategy based on sliding mode is proposed, which gives the good performance robust to disturbances as well as the fast transient responses.

The sliding mode control (SMC) is one of the popular strategies to deal with uncertain control systems [9]. The main feature of SMC is the robustness against parameter variations and external disturbances. Various applications of SMC have been conducted, such as robotic manipulators, aircrafts, DC motors, chaotic systems, and so on [10]. In this paper, a sliding mode controller is applied to the Buck DC-DC converter. It is shown via simulation results that the proposed controller has high performance both in the transient and in the steady state operations. A good control of the output voltage is obtained.

MODELLING OF THE BUCK DC-DC CONVERTER

A power circuit of the Buck DC-DC converter is introduced in figure 1. It consists of Tr, a controlled switch (IGBT), D, an uncontrolled switch (diode), L, an inductor, C, a capacitor and R, a load resistance. The switching of Tr is controlled by the pulse width modeling (PWM) feedback logic. The output voltage is controlled by comparing it with a reference voltage and using the error to adjust the duty ratio α . normally, the duty ratio is obtained by comparing the error or control voltage U_c with a fixed-frequency saw-tooth voltage V_{tr} . When U_c is greater than V_{tr} the switch is turned on, and consequently the diode is turned off. When U_c is less than V_{tr} the switch is turned off, and as a result the diode is turned on. In this case, the duty ratio in the n^{th} cycle is the solution of

$$U_c(\alpha(k)T) = V_{tr}(\alpha(k)T) \quad (1)$$

To derive a state-space representation for this converter operating in the continuous conduction mode with fixed-frequency duty-ratio control, we first treat the open loop system as a multi-topological system with two-circuit configurations. Each configuration describes the system in a sub-interval of time within the switching cycle T [13].

Hence, the period of each switching cycle can be divided into two time intervals: t_1 and t_2 , where $t_1 = \alpha \cdot T$ and $t_2 = T$. During t_1 , the switch is on and the diode is off, and during t_2 , the switch is off and the diode is on. In this model we assume that the diode and the switch are ideal. The inductor of the power stage has an equivalent series resistance r_L and the capacitor has an equivalent series resistance r_C . The inductor current is i_L and the output capacitor voltage is V_C . The topological sequence of the Buck converter consists of two linear time-invariant systems described by [13, 14].

Mode 1: $0 \leq t < t_1$

The inductor current ramps up during this mode of operation which is described by the following state-space equations:

$$\begin{cases} \dot{x} = A_1^o x + B_1^o V_e \\ V_s = C_1^o x \end{cases} \quad (2)$$

Where $x = [i_L, V_C]^T$, V_s is the sum of V_C and the voltage drop across r_C , V_e is the input voltage and the matrices A_1^o, B_1^o, C_1^o are given by [13, 14].:

$$A_1^o = \begin{bmatrix} -\frac{1}{L} \left(r_L + \frac{r_C \times R}{r_C + R} \right) & -\frac{1}{L} \left(\frac{R}{r_C + R} \right) \\ \frac{R}{C \times (r_C + R)} & -\frac{1}{C \times (r_C + R)} \end{bmatrix}, B_1^o = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}, C_1^o = \begin{bmatrix} \frac{r_C \times R}{r_C + R} & \frac{R}{r_C + R} \end{bmatrix}$$

Mode 2: $t_1 \leq t < t_2$

During this time, the inductor current falls and the system is described by the following state-space equations:

$$\begin{cases} \dot{x} = A_2^o x + B_2^o V_e \\ V_s = C_2^o x \end{cases} \quad (3)$$

where the matrices A_2^o, B_2^o, C_2^o are given by:

$$A_2^o = \begin{bmatrix} -\frac{1}{L} \left(r_L + \frac{r_C \times R}{r_C + R} \right) & -\frac{1}{L} \left(\frac{R}{r_C + R} \right) \\ \frac{R}{C \times (r_C + R)} & -\frac{1}{C \times (r_C + R)} \end{bmatrix}, B_2^o = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad (4)$$

$$C_2^o = \begin{bmatrix} r_c \times R & R \\ r_c + R & r_c + R \end{bmatrix}. \quad (5)$$

From mode 1 and mode 2 we deduce the following large-signal continuous-time system by taking the averages of the intervals and summing the results:

$$\begin{cases} \dot{\bar{x}} = (A_1^o \bar{x} + B_1^o \bar{V}_e) \alpha + (A_2^o \bar{x} + B_2^o \bar{V}_e) (1 - \alpha) \\ \dot{\bar{V}}_s = (C_1^o \alpha + C_2^o (1 - \alpha)) \bar{x} \end{cases} \quad (6)$$

Using the average model (4), we plot in figure 2 the inductor current and the capacitor voltage respectively. The solution of (2) and (3) is a discrete-time difference equation, which can be written in state-space form as:

$$x(k+1) = e^{A_2^o(1-\alpha(k))T} e^{A_1^o\alpha(k)T} x(k) \quad (7)$$

$$+ \begin{bmatrix} e^{A_2^o(1-\alpha(k))T} (e^{A_1^o\alpha(k)T} - I) (A_1^o)^{-1} B_1^o \\ (e^{A_2^o(1-\alpha(k))T} - I) (A_2^o)^{-1} B_2^o \end{bmatrix} V_e(k)$$

$$V_s(k+1) = C_2^o x(k+1) \quad (8)$$

Now, we introduce a PI controller which has this transfer function:

$$G_{contr}(p) = K_p \frac{1 + p.T_i}{p.T_i} \quad (9)$$

Where K_p and T_i are the parameters of the PI controller.

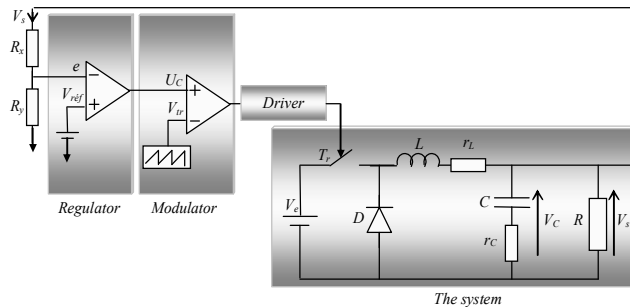


Fig1: Closed-loop buck regulator system with conventional P controller

Stability Analysis

The Buck DC-DC converter, presented in figure 2, employs a voltage feedback control loop. We plot in figure 3 the loop gain of the closed loop regulator system (with a proportional controller) for different values of the controller gain. The worst phase margin is 43° , and hence the converter is stable according to the small-signal averaged model [13, 14].

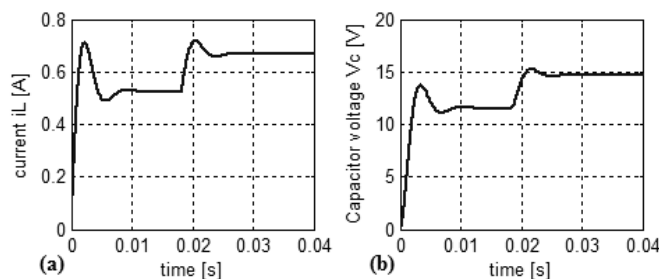


Fig 2: (a) Inductor current i_L , (b) Output capacitor voltage V_C

We show the bifurcation diagram where the bifurcation parameter is the controller gain. When $H_G < 8.4$, where H_G is the controller gain, the eigenvalues are inside the unit circle and hence the period-one solution is stable. As H_G is increased beyond 8.4, one of the Floquet multipliers exits the unit circle through -1, indicating a period doubling bifurcation. Consequently, the period-one solution loses stability

and the period of the response is doubled. After the period-doubling bifurcation the periodic 2 orbit directly bifurcates into a chaotic orbit.

SLIDING MODE CONTROL

Control applications of Buck DC-DC converters have been widely investigated [11]. The main objective of research and development in this field is always to find the most suitable control method to be implemented in various DC-DC converter topologies. Figure3 presents the control diagram of the presented SMC [14].

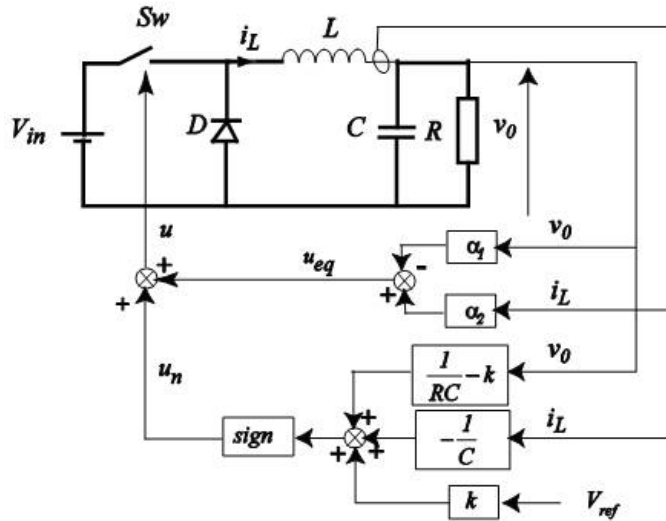


Fig 3:Sliding mode control for Buck converter

In other words, the goal is to select a control method capable of improving the efficiency of the converter, lessening the effect of electromagnetic interference (EMI), and being less effected by component variation which is the main objective in this work. The bloc scheme in figure 4 gives the configuration of the Buck DC-DC converter which utilizes a controller based on a sliding mode control law. In sliding mode control, the trajectory of the system is constrained to move or slide along a predetermined hyper plane in the state space (figure 5. (a) & (b)). Such mode is completely robust and independent of parametric variations and disturbances [12].

By eliminating the parasitic effect of the capacitor ($r_c = 0$), The system is described by the following state-space equations:

$$\dot{x} = F(x) + G(x, V_e) T_r \quad (10)$$

Where T_r is the switching function which can equal to 1 or 0, and the matrices F and G are given by:

$$F(x) = \begin{bmatrix} -\frac{r_L}{L} i_L - \frac{1}{L} V_c \\ \frac{1}{C} i_L - \frac{1}{RC} V_c \end{bmatrix}, \quad G(x, V_e) = \begin{bmatrix} \frac{V_e}{L} \\ 0 \end{bmatrix} \quad (11)$$

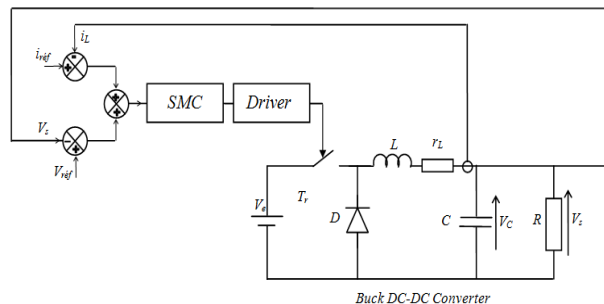


Fig. 4:Closed-loop Buck DC-DC converter with a sliding mode controller [14].

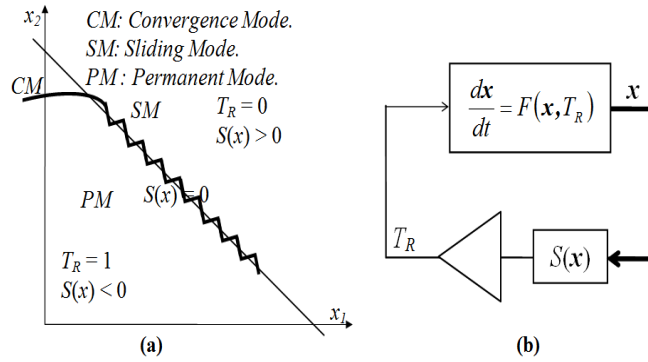


Fig. 5: (a). Trajectory modes in the state space, (b). Structure used of the SMC [14].

The general form of sliding surface which guarantees the convergence of the state x to its reference is given as follows:

$$S(x) = \left(\frac{d}{dt} + \lambda_x \right)^{r-1} e(x) \quad (12)$$

Where r is the degree of the sliding surface and λ is a strictly positive constant. It is the first convergence condition which permits dynamic system to converge towards the sliding surfaces. It is a question of formulating a positive scalar function $V(x) > 0$ for the system states variables which are defined by the following Lyapunov function:

$$V(x) = \frac{1}{2} S(x)^T S(x) \quad (13)$$

$$\dot{V}(x) < 0 \Rightarrow S(x)^T \dot{S}(x) < 0 \quad (14)$$

Now, to define the control algorithm, it contains two terms, first for the exact linearization, the second discontinuous one for the system stability.

$$T_R(t) = T_{Req}(t) + T_{Rn}(t) \quad (15)$$

Where $T_{Req}(t)$: is calculated starting from the expression $S(x) = 0 \Rightarrow \dot{S}(x) = 0$ and $T_{Rn}(t)$ is calculated from equation (15), it is given to guarantee the attractively of the variable to be controlled towards the commutation surface.

SIMULATION RESULTS

The PSPACE DS1104 controller board is a prototyping system. It is a real time hardware platform. It can be programmed with MATLAB/SIMULINK software through a real time interface allowing the generation of a real time code. Two ADC input channels of the DS1104, characterized by a 16 bits resolution, are used to acquire the Buck converter output voltage and the inductance current. The control board generates a digital PWM signal which is used to control the switch of the Buck converter. The proposed SMC was applied to a Buck converter characterized by the parameters given in the table 1.

Table 1: Studied buck converter parameters

Parameters	Values
V	30 V
C	15 μ F
L	5 mH
R	25 Ω
Switching frequency	25 kHz

To test the robustness of the SMC, we consider now the variations of the load resistance and the input voltage. Fig. 14 presents the evolution of the output voltage and the current in the load for the case of a sudden change of the load resistance from 30 Ω to 20 Ω . So by the application of the SMC, this perturbation was rejected in 10.10-3s and the output voltage attends the reference voltage after. Presented by figure 6, the output voltage evolution by application of the SMC to the studied Buck converter and figure7 the control signal evolution, and figure 8 the input voltage variation, and figure 9 the output voltage evolution by application of the SMC for the case of input voltage variation from 15 V to 10V.

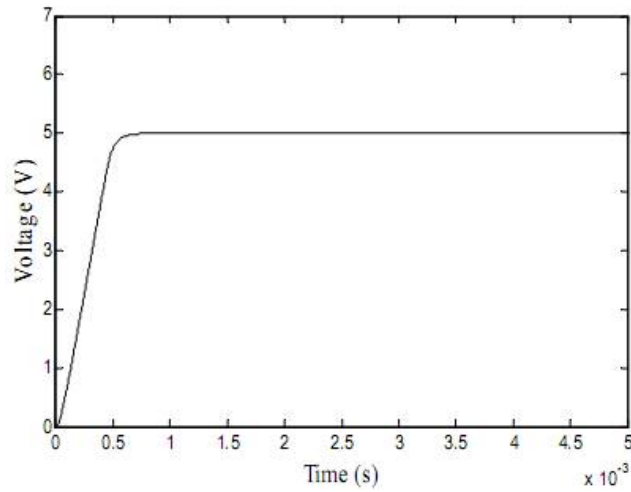


Fig. 6: Output voltage evolution by application of the SMC to the studied Buck converter

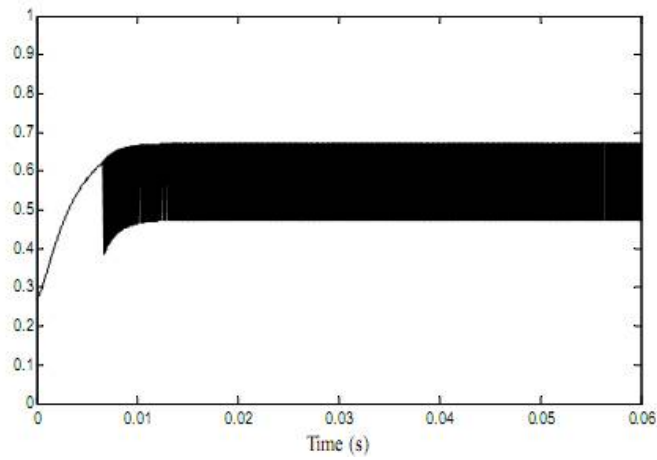


Fig. 7: Control signal evolution

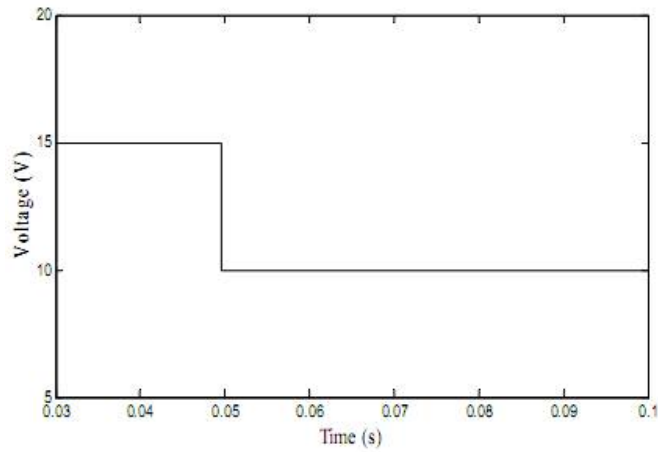


Fig. 8: Input voltage variation

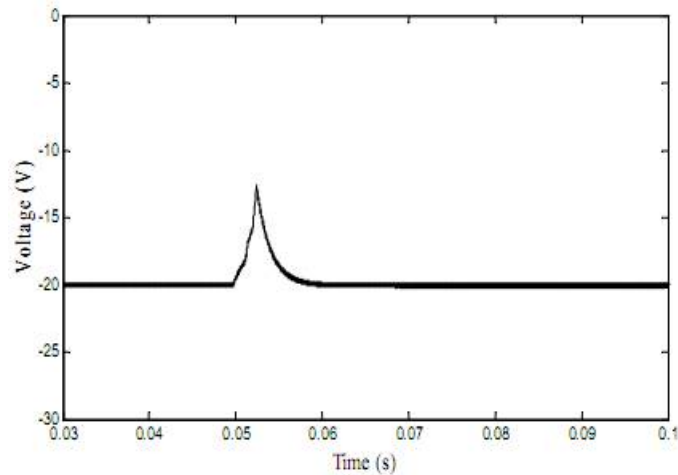


Fig. 9: Output voltage evolution by application of the SMC for the case of input voltage variation from 15 V to 10V

CONCLUSION

In this paper, Sliding Mode Control (SMC) for Buck, converters are proposed, tested and compared. SMC is suitable for switched mode DC-DC converters. Moreover, such control approach leads to good results. Two classical SMC are proposed respectively for Buck and Buck-Boost converters. The obtained simulation and practical results confirm the robustness of this control technique. paper a nonlinear model was derived for a Buck DC-DC converter. It consists of a discrete difference equation in addition to a switching constraint. Unlike approximate averaged or sampled-data models, this model enables the designer to investigate the behavior of the system in all regions of operation: stable and unstable. Equilibrium solutions were calculated and their stability was investigated. Then, bifurcation diagrams were generated to study the total behavior of the system as one of its parameters varies. Whereas the averaged model is capable of predicting the instabilities resulting from slow disturbances, it is incapable of predicting the instabilities resulting from the fast dynamics, such as sub harmonics and chaos. On the other hand, using the exact nonlinear model, one can predict the instabilities resulting from slow and fast disturbances. The exact model was used to study the stability of the Buck DC-DC converter with different compensation schemes (P, PI). Comparing the dynamic behaviors of this system, we found that instability exists in practical DC-DC converters even when the conventional design guidelines were followed. Finally, to improve the performances of our system, we have introduced a sliding mode controller.

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