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Residential consumers' optimal pricing based on Game theory

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ABSTRACT
Demand side management is of important subjects in Smart grids. Since the average cost of production due to changes in power consumption increases during the day, Time of Use Pricing (TOU) is a method for demand management for efficient consumer's management. In this paper, using Game theory and Stackelberg model, the most optimal kind of pricing among utility company and residential consumers may be introduced. The game pattern is set in such a way that utility company is leader and consumer is follower, so that the consumer determines his consumption with respect to the proposal of the optimal price by utility company. The cost function of utility have been proposed due to fluctuations in consumption. Also, a new cost function for consumers due to the difference between nominal demand and real consumption is presented, which according to the company's profits and consumer functions, the Nash equilibrium is achieved. In this paper, a scenario for residential consumers will be introduced according to a variety of Hourly, Flat and TOU pricing. According to the numerical results, it is observed that TOU is the most optimal pricing. In this case, the total amount of consumption and the average price per kWh of energy reduced and consumption shifts from peak hours to non-peak hours as well.
Keywords: *Smart grid, demand side management, residential consumer, TOU pricing, Game theory, Nash equilibrium point*

INTRODUCTION

With the restructuring of the power system, new concepts including Smart grids and demand side management have been emerged. The main objective of this program is to increase the efficiency and security of the electricity supply networks. Demand Response (DR) is among main methods of demand side management. This program is applied to motivate consumers to change consumption patterns in response to changes in market prices and is according to assist the operation indexes of the power network.

TOU is an effective method of demend response based on pricing which the utility companies, considering the different prices during the day are able to motivate consumers to reduce consumption during the day especially during peak load hours. In recent years, this type of pricing and especially Real-time pricing has been considered by many researchers. In [1], using the Stackelberg model and contrast between consumers, designing of residential consumption load through Real-time pricing was remarked which its results, not only lead to save money for consumers but also reduce consumption during peak load hours and cover the difference between demand and production as well.

In [2], an optimum model is introduced for consumption load with regard to Real-time pricing where the objective is to maximize the profit of utility due to a decrease in consumption during the day and consumption displacement during peak and non-peak load hours. In [3], according to the method of game theory and the contrast between the residential consumers, given Real-time pricing, the optimal consumption pattern is provided. While in Real-time pricing, prices will vary during the day hours, consumers can't quickly respond to it well and rapid response requires careful automation and knowledge about how to respond to the varying prices during the day. In [4], through an economic analysis on TOU pricing, it is showed that electricity demand has been stretched, so consumption in peak and non-peak hours, are partial alternatives. In [5], it has been shown in an experimental way that in addition to profitable being of pricing for utility but consumers are satisfied with the implementation of

this tariff. California's Statewide Pricing Pilot shows that residential, commercial and small industrial consumers as a result of varied pricing are willing to reduce electricity consumption at peak hours [6].
In this paper, according to the modeled satisfaction of consumer in [7], a new cost function based on self-elasticity [1]and the difference between actual consumption and contracted demand are provided and then, to make a game between the consumer and utility company, the most optimal pricing was determined from the perspective utility companies and consumers. The main goal of this paper, to reduce the overall consumption and the average fee paid per kWh and to transfer consumption from peak hours to non-peak hours; which is possible through TOU pricing compared with Flat and Hourly pricing methods.

1. Governing equations for the consumer

In this section, governing equations for consumers and the utility company are presented. The day is divided into N parts that for Hourly pricing: N = 24. The cost function of consumers includes the payment for electricity consumption as well as costs arising from the difference between the actual consumption and nominal demand, which is equivalent to:

(1)
$$C = \sum_{h=1}^{N} \rho_h l_h + \sum_{h=1}^{N} G_h(l_h, d_h)$$

ρ_h :the price of electricity sold at the h-th hour

l_h : The amount of actual load in response to the price in the h-th hour

d_h : The nominal demand of consumer in the h-th hour

$G_h(l_h, d_h)$: cost function of payment based on the difference between actual consumption and nominal demand h-th hour

Utility always with regard to the amount of consumers' consumption and contract demand gets money from them. In other words, whatever the consumption of subscribers be lower or higher than the contracted demand, they must pay a fee as penalty, because in both cases, utility based on contract demand generated electricity and if the consumption be less or more, the producer company will be affected. Certainly the amount of fines imposed on consumers with lower consumption of contract demand, is less than the fine of customers who have spent more than that. This amounts will be included in the bill as power fee and exceeding of demand cost.
According to the above explanation and satisfaction function proposed in [7], it is possible to define the paid fine by the consumer (G_h) based on l_h and d_h as follows:

(2)
$$G_h(l_h, d_h) = l_h \left| \beta_h \left(\frac{d_h}{l_h}\right)^{\alpha_h} - 1 \right|$$

(3)
$$\alpha_h = \left(\frac{1}{E_h}\right) + 1$$

(4)
$$\gamma_h = -\alpha_h \beta_h$$

In the above equations, E_h is the sign of consumers' self-elasticity in the hour h which it is always negative.
The range is between $\alpha_h < 1$ and $\alpha_h \beta_h < 0$.
In Table 1, the numerical changes of G_h , assuming a contract demand of 50 GW and self-elasticity of -0.4 for loads less or more than the contractual demand is shown that According to the table, numerical changes and the accuracy of the function G_h are evident .

Table 1. Numerical changes of function G_h

Contract demand	Actual consumption	Self-elasticity	factor α **Error! Bookmark not**	factor β **Error! Bookmark not**	Numerical value

[1] The relationship between change of price and change of demand can be described as $(\partial d/d)/(\partial \rho/\rho)=\varepsilon$, where ε is the price elasticity [7].

(d)	(L)	(E)	**defined.**	**defined.**	of G_h
50	30	-0.4	-1.5	6.66667	107.48
50	45	-0.4	-1.5	6.66667	43.85
50	50	-0.4	-1.5	6.66667	0
50	60	-0.4	-1.5	6.66667	125.82
50	150	-0.4	-1.5	6.66667	4196.15

Variation diagram of the function G_h based on $\frac{d_h}{l_h}$ and given various parameters such as β_h and α_h in Figure 1 is shown. Due to the shape, this function is responsive for different values of the elasticity of demand and thus is applied for different types of consumers with different elasticities.

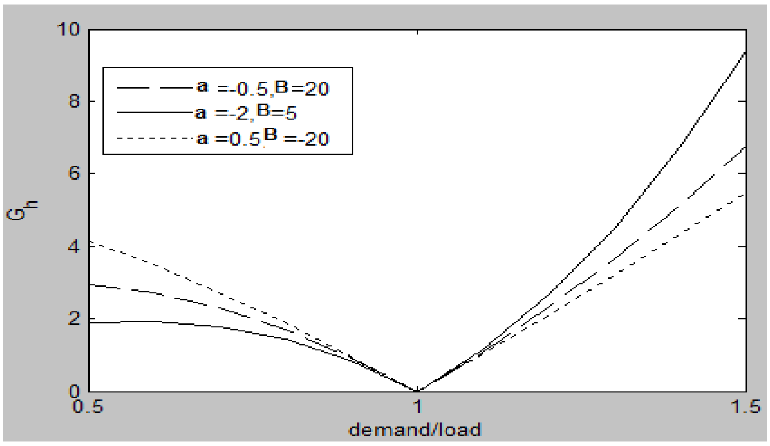

Figure 1: Variation diagram of the function G_h based on various parameters of β_h and α_h

Thus, according to (1), the consumer's benefit function is equal to:

(5)

$$u_1 = -C = -\sum_{h=1}^{N} (\rho_h l_h + G_h)$$

The profit of utility is modeled as follows:

(6)

$$u_2 = \sum_{h=1}^{N} \rho_h l_h - \sum_{h=1}^{N} c_h g_h + \sum_{h=1}^{N} G_h(l_h, d_h) - f(g)$$

c_h is the marginal cost of electricity in the h-th hour and g_h is the power supplied in the h-th hour

The function $f(g)$ indicates the cost which based on changes in consumer consumption imposed on the distribution company. This cost as the sum of squared deviations which is multiplied by a μ coefficient, is defined as follows [7]:

$$f(g) = \mu \sum_{h=1}^{N} (g_h - \bar{g})^2 \qquad (7)$$

In the above function, $\bar{g}_h$ indicates the average power generated during a day.

Considering the equality $l_h = g_h$, equation (6) can be rewritten as follows

(8)

$$u_2 = \sum_{h=1}^{N} \rho_h l_h - \sum_{h=1}^{N} c_h l_h + \sum_{h=1}^{N} G_h(l_h, d_h) - f(l)$$

Model of Game Theory

The purpose of this paper is to find the most optimal pricing due to maximize utility company profits and consumer simultaneously, using Game theory. The objective function is formulated as follows:

(9)

$$\rho^* = \arg\max_{\rho} \ u_2 = \sum_{h=1}^{N} (\rho_h l_h - c_h l_h + G_h) - f(l)$$

(10)

$$l^* = \arg\max_{l} \ u_1 = -\sum_{h=1}^{N} (\rho_h l_h + G_h)$$

Restrictions intended for this function include:

1. $l_{h_{\min}} \le l_h \le l_{h_{\max}} = \min(d_{h,\max}, g_{h,\max})$
2. $c_h \le p_h$

To provide the minimum load required for the consumer, the electricity production company should not consider a high price, hence, a lower limit is set for the consumer. Also, real and consuming load shall not exceed $\min(d_{h,\max}, g_{h,\max})$ and this means that consumers tend, at a specific time interval, increase their consumption, but the amount shall not exceed the maximum production and be limited to increase consumption. The second limitation is to guarantee that the production cost of electricity is always greater than or equal to the costs of selling it [7].

Since competition in the electricity market is incomplete, as a result, the decisions of market participants including consumers and the electricity company influence each other and when the market reaches to its equilibrium and optimal point that Nash points of the game be obtained. When obtaining the Nash points (ρ^*, l^*) is guaranteed that the following conditions exist:

(11)

$$l \ne l^* : u_1(\rho^*, l^*) \ge u_1(\rho^*, l)$$

(12)

$$\rho \ne \rho^* : u_2(\rho^*, l^*) \ge u_2(\rho, l^*)$$

According to the game of Stackelberg, consumer as follower and the utility company as leader are considered. Accordingly, first maximize u_1 with respect to $\{l_h\}_{h=1}^{N}$ and the optimal response of $l^*(\rho)$ in u_2 is placed and optimum it given $\{\rho_h\}_{h=1}^{N}$ .Process is as follows:

To obtain the optimum reaction of load against for the adopted price of the electricity company, electricity prices should be considered in different time periods and (5) were derived based on l_h as the result of which is:

(13)

$$if : l_h < d_h \rightarrow l_h^*(\rho_h) = d_h \sqrt[\alpha_h]{\frac{\beta_h (1 - \alpha_h)}{\rho_h + \beta_h}}$$

$$if : l_h > d_h \rightarrow l_h^*(\rho_h) = d_h \sqrt[\alpha_h]{\frac{\beta_h (\alpha_h - 1)}{\rho_h - \beta_h}}$$

So the placement obtained l_h^* from (13) in (8), ρ_h^* can be obtained:

(14)

$$u_2(\rho) = \sum_{h=1}^{N} \rho_h l_h^*(\rho_h) - \sum_{h=1}^{N} c_h l_h^*(\rho_h) + \sum_{h=1}^{N} G_h(l_h^*(\rho_h), d_h) - f(l_h^*(\rho_h))$$

In this paper, the above equations for consumers of residential types have been studied. Residential consumers are usually price-sensitive and willing to change their consumption patterns according to price changes at different times. The flexibility of these consumers is relatively low and their ability to increase or decrease their total electricity consumption is limited but instead, they tend very much to plan and schedule their consumption to reduce the cost of their payment [7].

Simulation

In this paper, the below numerical examples are applied to assess the presented methods: In this example, as in Table 2, the day is divided into 4 blocks of time [7].

Table 2: Time intervals for TOU pricing

period	Time intervals (hours)
Peak	From 2pm to 7pm
Semi-peak	From 5 am to 2 pm- from 7pm to 12 pm
Off peak	From 12 pm to 5 am

Values of consumers' nominal demand (d_h) per hour and consumers' self-elasticity per hour (E_h) respectively, are shown in the figures (2) and (3). According to the experiment of Ameren Illinois, a report on the hourly pricing of electricity for 11,000 domestic consumers during the 4 years is presented [10]. Data related to the elasticity and marginal cost of production have been obtained from [7] and [11]. They estimated the elasticity range and the E_h in this paper is selected based on them.

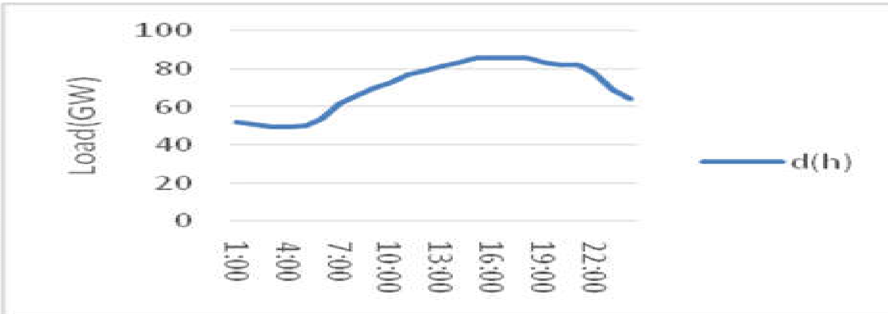

Figure 2: the diagram of the hourly nominal values of consumer

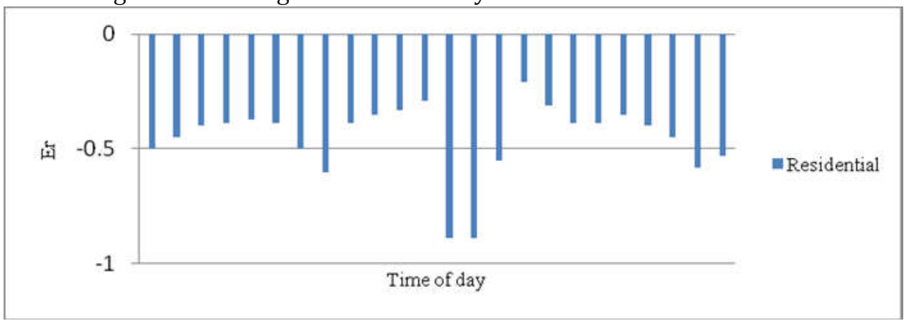

Figure 3: self- elasticity of consumer's demand per hour (E_h)

It should be noted that contract demand is considered equal to the nominal demand. Also, it is assumed that system capacity is 1.1 of maximum nominal demand of the consumer. Also, $l_{h,\min}$ and $l_{h,\max}$ are respectively considered as 90 % and 125 % of nominal consumer demand. According to the assumption, γ_h and μ are equal to 10 and 1, respectively. [7]

Total load of consumers and the average price per kWh of energy are:

$$l_{total} = \sum_{h=1}^{N} l_h \qquad (15)$$

$$\bar{\rho} = \frac{\sum_{h=1}^{N} \rho_h l_h}{\sum_{h=1}^{N} l_h} \qquad (16)$$

To solve the problem the Software GAMS was used and because of the nonlinearity the Conopt solver was applied. Based on the input data and the introduced Game theory model, the optimal pricing and consumers' consumption strategy is obtained. Figures (4) and (5) respectively, show the price and household consumption during pricing as Flat, Hourly and TOU.

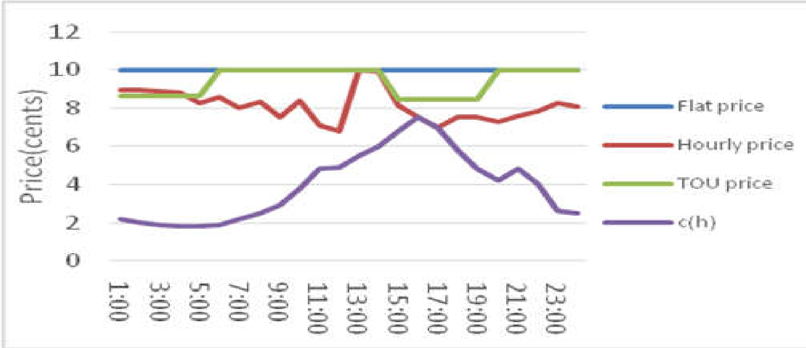

Figure 4: Comparing the price based on the Flat, Hourly and TOU pricing strategies

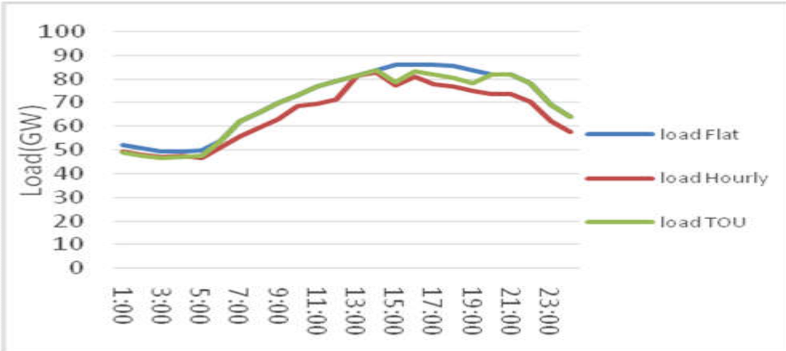

Figure 5: comparing the consumption based on the Flat, Hourly and TOU pricing strategies

Given the above, it can be seen that to reduce the payment, the residential consumer at the TOU pricing strategy, shifts its electricity consumption from peak hours to non-peak hours.
In Table 3, the results of the simulation scenario have been collected in terms of Flat, Hourly and TOU pricing. As can be seen, from the perspective of the electricity company, the Flat pricing is more profitable, however, in this method, the total amount of consumption and expenditure are increased and in this respect, this type of pricing in the electricity market is not appropriate. At the TOU pricing strategy, the price paid per kWh of energy is lower than other pricing ways and therefore consumers' cost is saliently lower. Also, given that there is no salient difference between the electricity company's profits in TOU pricing and the Hourly pricing, due to the price paid per kWh of energy, it can be stated that applying the TOU pricing method is the most optimal pricing.

Table 3: The results obtained from scenario simulation

Parameters obtained from simulation	**TOU**	**Flat**	**Hourly**
The profit of electrical company (M $)	26827.258	28582.280	26062.979
Total cost paid by consumers (M $)	38110	40070	67310
Total consumption (GWh)	1660.439	1700.1	1567.466
$\overline{\rho_R}$ (¢/KWh)	9.428	10	16.188

CONCLUSION
In this paper, based on self-elasticity, a cost function was introduced for consumer and accordingly, the game between the utility company and the consumer was simulated in the form of a scenario for the residential consumer. Simulations based on three pricing methods (Hourly, Flat and TOU) was done and by comparing the results, the most optimal pricing method has been determined. According to the results, it is seen that at the status of pricing based on TOU, the average price paid per kWh of energy is lower than the average price paid by the consumer in any other pricing methods and as a result, consumers pay

much less. Also at the method of TOU, the consumer to reduce his cost, reduced power consumption at peak hours and transfer it to non-peak hours. Therefore, it can be stated that the optimal pricing method for of the utility company is pricing based on TOU.

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