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Robust Control of a Quadrotor using EKBF State Vector Estimation

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ABSTRACT
This paper presents the design of an optimal nonlinear controller for trajectory tracking of a quadrotor helicopter based on the well-known sliding mode control method (SMC). To this end, the Lyapunov law analysis has been used regarding nonlinear equations of the quadrotor. The proposed control method is characterized by its robustness against disturbances and aerodynamic effects. One of the main targets of this study is to develop a nonlinear observer based on extended kalmanbucy filter (EKBF) to estimate unmeasured states of the system. In order to optimize the parameters in proposed control method, the paper utilizes the genetic algorithm. Finally, simulation results indicate that quadrotor UAV with the proposed controller ensures good tracking of a desired trajectory and its robustness against aerodynamic effects is better than the other previously presented works.
Keywords— Quadrotor, Sliding Mode Control, Genetic Algorithm, State Estimation, Extended KalmanBucy Filter

INTRODUCTION

The quadrotor is an unmanned aerial vehicle (UAV) which has four propellers attached to four motors placed on a fixed body to create six strongly coupled output coordinators [1,2]. Nowadays, these UAVs are vastly used in various applications at minimal cost and without endangering any risk to human life. UAVs are generally suitable for military applications, cartography, surveillance and acquisition of targets. In [3,4] authors describe the aerodynamic forces and moments on rotor dynamic of helicopter by synthesizing momentum with blade element theory. In [6] the SMC has been applied extensively to control quadrotors. A robust second order sliding mode control (SMC) for controlling a quadrotor with uncertain parameters presented based on high order sliding mode control (HOSMC) for trajectory tracking of a quadrotor helicopter. [7] presents the sliding mode control of a class of under actuated systems regarding the quadrotor as a sample model. In [8] the authors use a continuous sliding mode control method based on feedback linearization applied to a quadrotor UAV. [9] gives a new robust backstepping-based controller that induces integral sliding modes for the under actuated dynamic model of a quadrotor subject to smooth bounded disturbances, consisted of wind gust and aerodynamics effects. In [10-11] the authors present a design method for attitude control of a quadrotor based on sliding mode control method in which all sliding surfaces are designed using the Lyapunov stability theory. Other methods have also been attempted to achieve good performance [12-14].

In a specific problem of a continuous-time stochastic system, inorder to improve the results, state estimation with a nonlinear filter such as Extended Kalman-Bucy Filter (EKBF) is used which constitutes the basis for [15]. [16] designed an EKBF for accurate and robust tracking of targets in an air combat scenario. A comparative study on applying modern techniques such as Genetic Algorithm (GA) for parameter optimization of controllers is presented in [17]. In [18-20] the effectiveness of this technique has been shown in position control and path generation of robotic systems. In [21] parameter optimization was obtained for a small helicopter based on GA focusing on stability of the system.

In this paper, we utilize the EKBF method along with nonlinear dynamics of the quadrotor as an efficient solution to estimate unmeasured states of the quadrotor dynamics. Finally, a nonlinear robust controller is developed with optimized control parameters adjusted by GA for decreasing the impact of aerodynamic effects on modeling and control of the system. The paper is organized as follows: Section 2 presents the dynamic model of a quadrotor. In section 3 the presentedEKBF is described. Section 4 proposes the developed sliding mode control technique for the quadrotor along with an optimal method using Genetic

Algorithm which ensures the locally asymptotic stability of the system. Finally the simulation results have been presented in section 5 and some concluding remarks (section 6) end the paper.

2.Dynamical model of quadrotor

Quadrotor is anextremelynonlinear, multivariable, strongly coupled, and under actuated system (six degrees of freedom with only four actuators) as shown in Fig.1. $F_i \ (i = 1,2,3,4)$ in the figure is the trust force produced by rotors.

Simultaneous increase or decrease in speed of all rotors, will generate the vertical motion of the quadrotor. To analyze the dynamics of this system, consider two reference frames.The earth-fixed frame and the body-fixed reference framedenoted by $E = \{E_x, E_y, E_z\}$ and $B = \{B_x, B_y, B_z\}$ respectively. The position $\zeta = [x, y, z]^T$ and the angle $\Theta = [\varphi, \theta, \psi]^T$ are defined in the reference frame E. These three Euler Angels are named roll angle $(-\pi/2 \prec \varphi \prec \pi/2)$ pitch angle $(-\pi/2 \prec \theta \prec \pi/2)$ and yaw angle $(-\pi \prec \psi \prec \pi)$.

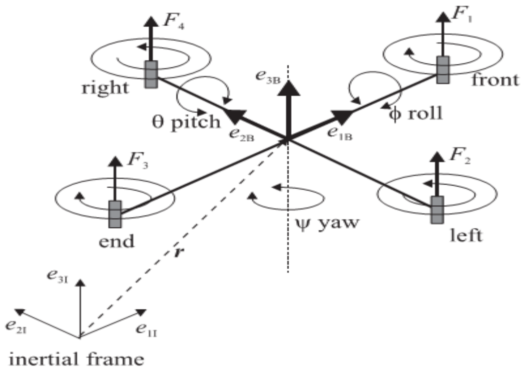

Fig. 1.Quadrotor helicopter schematic

The rotation matrix from B to E is:

$$R(\Omega) = \begin{bmatrix} C_\psi C_\theta & C_\psi S_\theta S_\varphi - S_\psi C_\varphi & C_\psi S_\theta C_\varphi + S_\psi S_\varphi \\ S_\psi C_\theta & S_\psi S_\theta S_\varphi - S_\psi C_\varphi & C_\psi S_\theta C_\varphi + S_\psi S_\varphi \\ -S_\theta & C_\theta S_\varphi & C_\theta S_\varphi \\ & & \end{bmatrix} \qquad (1)$$

Rotor aerodynamics:

The aerodynamic forces and moments are established by synthesizing momentum with blade element theory according to results of [3-4]. The power exertedon each motor produces a torque, M_{Qi},which results in a thrust F_i thatcan be derived as:

$$\begin{bmatrix} F_i \\ M_{Qi} \end{bmatrix} = \begin{bmatrix} C_F \rho A r^2 \Omega^2 r \\ C_Q \rho A r^2 \Omega^2 r \end{bmatrix} \qquad (2)$$

where aerodynamic coefficients are denoted by thrust coefficient C_F and torque coefficient C_Q. Also, A is the area of propeller disk, $\rho = 1.293 \ kg/m^3$ is the air density, r is the radius of the blade and Ω is the angular velocity of the quadrotor propeller. The generated moment about the roll axis, M_{Ri}, and drag force on the rotor, D_i, can also be given as:

$$\begin{bmatrix} M_{Ri} \\ D_i \end{bmatrix} = \begin{bmatrix} C_R \rho A r^2 \Omega^2 \\ C_D \rho A r^2 \Omega^2 \end{bmatrix} \qquad (3)$$

where aerodynamic coefficients are denoted by drag coefficient C_D and roll coefficient C_R.

State Space Model of the Quadrotor

The full quadrotor dynamic model without aerodynamics effects, regarding the x, y, z motions as outcome of pitch, roll or yaw rotationscan be written as:

$$\begin{cases} \ddot{x} = (\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi)\dfrac{l}{m}U_1 \\ \ddot{y} = (\cos\phi \sin\theta \sin\psi - \sin\phi \sin\psi)\dfrac{l}{m}U_1 \\ \ddot{z} = -g + (\cos\phi \cos\theta)\dfrac{l}{m}U_1 \\ \ddot{\varphi} = \dot{\theta}\dot{\psi}\left(\dfrac{I_y - I_z}{I_x}\right) - \dfrac{j_r}{I_x}\dot{\theta}\Omega + \dfrac{l}{I_x}U_2 \\ \ddot{\theta} = \dot{\phi}\dot{\psi}\left(\dfrac{I_z - I_x}{I_y}\right) - \dfrac{j_r}{I_y}\dot{\phi}\Omega + \dfrac{l}{I_y}U_3 \\ \ddot{\psi} = \dot{\phi}\dot{\theta}\left(\dfrac{I_x - I_y}{Iz}\right) + \dfrac{l}{I_z}U_4 \end{cases}$$

(4)

Table 1 in section 5, summarizes different parameters of the prototype quadrotor. The inputs of the system posed on U_1, U_2, U_3, U_4 and Ω as a disturbance are given by:

$$\begin{cases} U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ U_2 = b(\Omega_4^2 - \Omega_2^2) \\ U_3 = b(\Omega_3^2 - \Omega_1^2) \\ U_4 = d(\Omega_2^2 - \Omega_1^2 + \Omega_4^2 - \Omega_3^2) \\ \Omega = \Omega_2 + \Omega_4 - \Omega_1 - \Omega_3 \end{cases} \qquad (5)$$

The model (4) can be rewritten in state-space form $\dot{X} = f(X, U)$ by introducing $X = [x_1, \ldots\ldots, x_{12}]$ as the state vector of the system:

$$\begin{cases} x_1 = x & , x_7 = \varphi \\ x_2 = \dot{x}_1 = \dot{x} & , x_8 = \dot{x}_7 = \dot{\varphi} \\ x_3 = y & , x_9 = \theta \\ x_4 = \dot{x}_3 = \dot{y} & , x_{10} = \dot{x}_9 = \dot{\theta} \\ x_5 = z & , x_{11} = \psi \\ x_6 = \dot{x}_5 = \dot{z} & , x_{12} = \dot{x}_{11} = \dot{\psi} \end{cases} \qquad (6)$$

In the remaining of this section the translational and rotational dynamic equations with aerodynamics effects according to Newton-Euler method are derived. Firstly the translational dynamic equation is given by:

$$F_{total} = m\ddot{\xi} \qquad (7)$$

where, F_{total} is the external force which can be defined as:

$$F_{total} = F_{rotor} - F_{aero} - F_G \qquad (8)$$

in which $F_G = mG$ is the gravitational force, $G = [0, 0, g]^T$ and F_{rotor} indicates the aerodynamic force of the rotor and F_{aero} is the air resistance:

$$F_{rotor} = R(\Omega)(\sum_{i=1}^{4} F_i - \sum_{i=1}^{4} D_i)$$

$$F_{aero} = \frac{1}{2}\rho AC(U^B)^2 \qquad (9)$$

where C is aerodynamic force coefficient, $C = diag[C_x, C_y, C_z]$.

Now, from eq. (9) the translational dynamicequations are defined [22]:

$$\begin{cases} \ddot{x} = \frac{1}{m}[(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)\Sigma_{i=1}^{4} F_i - \\ \Sigma_{i=1}^{4} D_{xi} - \frac{1}{2}\rho AC_x(U_x{}^B)^2] \\ \ddot{y} = \frac{1}{m}(\cos\phi\sin\theta\sin\psi - \sin\phi\sin\psi)\Sigma_{i=1}^{4} F_i - \\ \Sigma_{i=1}^{4} D_{yi} - \frac{1}{2}\rho AC_y(U_y{}^B)^2] \\ \ddot{z} = -g + \frac{1}{m}[(\cos\phi\cos\theta)\Sigma_{i=1}^{4} F_i - \frac{1}{2}\rho AC_z(U_z{}^B)^2] \quad (10) \end{cases}$$

Next the rotational dynamic equations are obtained [22]:

$$M_{total} = \Theta + \ddot{\Theta} \times (I\dot{\Theta})$$

$$M_{total} = M_c + M_g + M_R \qquad (11)$$

Where $M_g = \Theta\Omega j_r(-1)^{i+1}\sum_{i=1}^{4} M_{gi}$ is the gyroscopic torque of the rotors. $M_R = (-1)^{i+1}\sum_{i=1}^{4} M_{Ri}$, is the rolling moment and M_c is the moment generated by rotors:

$$M_C = \begin{bmatrix} l(-F_2 + F_4) \\ l(-F_1 + F_3) \\ (-1)^{i+1}\sum_{i=1}^{4} M_{Qi} \end{bmatrix} \qquad (12)$$

Now, from Eqs. $(11-12)$ we have:

$$\begin{cases} \ddot{\varphi} = \dot{\theta}\dot{\psi}\left(\dfrac{I_y - I_z}{I_x}\right) - \dfrac{j_r}{I_x}\dot{\theta}\Omega + \dfrac{l}{I_x}(-1)^{i+1}\sum_{i=1}^{4} M_{Rxi} \\ + \dfrac{l}{I_x}(F_4 - F_2) \\ \ddot{\theta} = \dot{\phi}\dot{\psi}\left(\dfrac{I_z - I_x}{I_y}\right) - \dfrac{j_r}{I_y}\dot{\phi}\Omega + \dfrac{l}{I_y}(-1)^{i+1}\sum_{i=1}^{4} M_{Ryi} \\ + \dfrac{l}{I_Y}(F_1 - F_3) \\ \ddot{\psi} = \dot{\phi}\dot{\theta}\left(\dfrac{I_x - I_y}{Iz}\right) + -\dfrac{j_r}{I_Z}\dot{\varphi}\Omega + \dfrac{l}{I_z}(-1)^{i+1}\sum_{i=1}^{4} M_{Qi} \end{cases} \quad (13)$$

finally, the general state space equation of quadrotor using state variables in eq.(6) can be written as:

$$f(X,U) = \begin{pmatrix} x_2 \\ u_x \dfrac{1}{m} U_1 \\ x_4 \\ u_y \dfrac{1}{m} U_1 \\ x_6 \\ -g + (\cos x_7 \cos x_9)\dfrac{1}{m} U_1 \\ x_8 \\ x_{10} x_{12} a_1 + x_{10} a_2 \Omega + b_1 U_2 \\ x_{10} \\ x_8 x_{10} a_3 + x_8 a_4 \Omega + b_2 U_3 \\ x_{12} \\ x_8 x_{10} a_5 + b_3 U_4 \end{pmatrix} \quad (14)$$

where,

$$\begin{aligned} & a_1 = (I_y - I_z)/I_x \\ & a_2 = -j_R / I_x \qquad ,b_1 = l/I_x \\ & a_3 = (I_z - I_x)/I_y \ , \ \ b_2 = l/I_y \qquad (15) \\ & a_4 = -j_R / I_y \qquad ,b_3 = l/I_z \\ & a_5 = (I_x - I_y)/I_z \end{aligned}$$

$$\begin{aligned} & u_x = (\cos x_5 \sin x_7 \cos x_9 + \sin x_5 \sin x_7) \\ & u_y = (\cos x_5 \sin x_7 \sin x_9 - \sin x_5 \sin x_7) \end{aligned} \quad (16)$$

The variables u_x and u_y can be considered as virtual commands whichrotate the thrust vector U_4 in such a way that the desired $x - y$ translational motion is achieved.

Extended KalmanBucy Filter

The Kalmanfilter provide same astonier missing information from in direct (and noisy) measurements. It also provides optimal (minimum variance) state estimation when the dynamic system is linear. The EKBF is an optimal recursive estimation algorithm for computing the states of a nonlinear stochastic system with uncorrelated Gaussian process and measurement noise. According to eq.(6), the states vector is then defined as:

$$X = [x, \dot{x}, y, \dot{y}, z, \dot{z}, \varphi, \dot{\varphi}, \theta, \dot{\theta}, \psi, \dot{\psi}] \qquad (17)$$

Let the state and output equations of a Gaussian stochastic system be described by the following stochastic differential equations [15,16]:

$$\begin{aligned} \dot{x} &= f(x(t), u(t), t) + \omega(t), \quad \omega(t) \sim N(0, Q(t)) \\ y(t) &= h(x(t), t) + v(t), \quad v(t) \sim N(0, R(t)) \end{aligned} \qquad (18)$$

where $x(t)$ is state vector and $y(t)$ is the measurements vector. $\omega(t)$ is the state noise and the vector $v(t)$ is the measurement noise. The state noise and the measurement noise are mutually independent and independent of the initial state x_0 whose probability density $p(x_0)$ is determined.Then the initial conditions are:

$$\hat{x}(t) = E[x(t_0)], \;\; P(t_0) = \mathrm{var}[x(t_0)] \qquad (19)$$

in which $\hat{x}(t)$ is the estimate of x vector.

When the system is linear with assumption of Gaussian probability densities for the states and noises, the functions $f(x(t), t)$ and $h(x(t), t)$ are linear with respect to the states, and the solution for the estimation problem is the Kalman-Bucy Filter.However, if the system is nonlinear, the probability densities may be approximated by Gaussian densities such that their moments can be recursively evaluated by a set of approximately linear equations which define the Extended Kalman Bucy Filter [15,16].

In prediction step, the mean $\hat{x}(t)$ and covariance matrix $p(t)$ for a continuous interval satisfy the following ordinary differential equations:

$$\dot{\hat{x}} = f(\hat{x}(t), u(t)) + K(t)(y(t) - h(\hat{x}(t)))$$

$$\dot{P}(t) = F(t)P(t) + P(t)F(t)^T - K(t)H(t)P(t) + Q(t)$$

$$K(t) = P(t)H(t)R(t)^{-1} \qquad (20)$$

where $F(\hat{x}(t), t)$ is the Jacobean matrix of $f(x(t), t)$ evaluated at $\hat{x}(t)$ and $H(\hat{x}(t), t)$ is the Jacobean matrix of $h(x(t), t)$ evaluated at $\hat{x}(t)$:

$$F(t) = \left. \frac{\partial f(x(t), u(t), t)}{\partial x} \right|_{\hat{x}(t), u(t)}$$

$$H(t) = \left. \frac{\partial h}{\partial x} \right|_{\hat{x}(t)} \qquad (21)$$

Therefore, according to eq.(21), in this study the Jacobean matrix $H(\hat{x}(t), t)$ and $F(\hat{x}(t), t)$ are set as :

$$F(\hat{x}(t), t) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_1\hat{x}_6 & 0 & a_1\hat{x}_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_3\hat{x}_6 & 0 & 0 & 0 & a_3\hat{x}_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_5\hat{x}_6 & 0 & a_5\hat{x}_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$H(\hat{x}(t), t) = [I_{12\times12}] \qquad (22)$$

and $Q(t)$ is noise power measurement and $R(t)$ is the covariance matrix of the state noise measurement that are given by:

$$R = diag\,(\mathrm{var}(v))$$

$$Q = diag\,(\mathrm{var}(\omega)) \qquad (23)$$

In Extended KalmanBucy Filter choosing the value of R and Q matrices influence on the system states. In this regard, choice of the noise power is very substantial such that it could be very effective in reducing the settling time to stability of the system. In kalman filter the value of P covariance's matrixmust not go to zero during the yield action, so define an initial condition for that in simulation steps, for this target describe a lower limit to so this measure replace by it. The measurement values of x, y and h are obtained from the measurement equation of the linear type $y(t) = h(x(t), t) + v(t)$.

Sliding Mode Control of the Quadrotor

Indicates the $\hat{X}$ is estimation of state vector (17)with:

$$\hat{X} = [\hat{x}_1, ..., \hat{x}_{12}]^T = [\hat{x}, \hat{\dot{x}}, \hat{y}, \hat{\dot{y}}, \hat{z}, \hat{\dot{z}}, \hat{\varphi}, \hat{\dot{\varphi}}, \hat{\theta}, \hat{\dot{\theta}}, \psi, \hat{\dot{\psi}}]$$

The sliding mode controller designing procedure is exerted in two steps. Firstly, choice of sliding surface (S) is produced according to the tracking error, In second step consider a Lyapunov function that verified the necessary condition for stability ofLyapunov law [12-14].The sliding mode control to quadrotor state variables estimated dynamic is presented by establishing the statement for control input. The sliding surfaces are described as follows:

$$\begin{cases} s_x = e_2 + \alpha_1 e_1 \\ s_y = e_4 + \alpha_2 e_3 \\ s_z = e_6 + \alpha_3 e_5 \\ s_\varphi = e_8 + \alpha_4 e_7 \\ s_\theta = e_{10} + \alpha_5 e_9 \\ s_\psi = e_{12} + \alpha_6 e_{11} \end{cases} \quad (24)$$

Such that $\alpha_i \succ 0$ and $\begin{cases} e_i = x_{id} - \hat{x}_i \\ e_{i+1} = \dot{x}_{id} - \hat{x}_{i+1} \end{cases}, i \in [1, 11]$ Assuming here that $V(s_\varphi) = \frac{1}{2} s_\varphi^2$ then, the necessary sliding condition is surveyed and Lyapunov stability is guaranteed. The chosen law for the attractive surface is the time derivative of $V(s_\varphi)$ satisfying $s_\varphi \dot{s}_\varphi \prec 0$

$$\begin{aligned} \dot{s}_\varphi &= k_1 sign(s_\varphi) \\ &= \dot{e}_8 + \alpha_4 \dot{e}_7 \\ &= \ddot{x}_{7d} - \hat{\dot{x}}_8 + \alpha_3 (\dot{x}_{7d} - \hat{x}_8) \\ &= -\hat{x}_{10} \hat{x}_{12} a_1 - \hat{x}_{10} a_2 \Omega - b_1 U_2 + \ddot{\varphi}_d \\ &+ \alpha_4 (\dot{\varphi}_d - \hat{x}_8) \end{aligned} \quad (25)$$

According to the control equation of roll angle (φ) in eq. (14), control U_2 is obtained:

$$\begin{aligned} U_2 &= \frac{1}{b_1}(-k_1 sign(s_\varphi) - \hat{x}_{10}\hat{x}_{12}a_1 - \hat{x}_{10}a_2\Omega \\ &+ \ddot{\varphi}_d + \alpha_4(\dot{\varphi}_d - \hat{x}_8)) \\ &= \frac{1}{b_1}(-k_1 sign(s_\varphi) - \hat{x}_{10}\hat{x}_{12}a_1 - \hat{x}_{10}a_2\Omega \\ &+ \ddot{\varphi}_d + \alpha_4 e_8) \end{aligned} \quad (26)$$

Similarly, pitch angle (θ) control input U_3 can be obtained as:

$U_3 = \frac{1}{b_2}(-k_2 sign(s_\theta) - \hat{x}_8\hat{x}_{12}a_3 - \hat{x}_8 a_4 \Omega + \ddot{\theta}_d + \alpha_5 e_{10})$ and yaw angle (ψ) control input U_4 is written as:

$$U_4 = \frac{1}{b_3}(-k_3 sign(s_\psi) - \hat{x}_{10}\hat{x}_8 a_5 + \ddot{\psi}_d + \alpha_6 e_{12})$$

Also altitude control input U_1 and the control variables U_x and U_y can be obtained:

$$U_1 = \frac{m}{\cos\hat{x}_7 \cos\hat{x}_9}(-k_4 sign(s_z) + g + \ddot{z}_d + \lambda\alpha_3 e_6)$$

$$U_x = \frac{m}{U_1}(-k_5 sign(s_x) + \ddot{x}_d + \alpha_1 e_2)$$

$$U_y = \frac{m}{U_1}(-k_6 sign(s_y) + \ddot{y}_d + \alpha_2 e_4) \qquad (27)$$

Finally, from eq. (24) the sliding surfaces are chosen as:

$$\begin{cases} s_x = e_2 + \alpha_1 e_1 = \dot{x}_{1d} - \hat{x}_2 + \alpha_1(x_{1d} - \hat{x}_2) \\ s_y = e_4 + \alpha_2 e_3 = \dot{x}_{3d} - \hat{x}_4 + \alpha_2(x_{3d} - \hat{x}_4) \\ s_z = e_6 + \alpha_3 e_5 = \dot{x}_{5d} - \hat{x}_6 + \alpha_3(x_{5d} - \hat{x}_6) \\ s_\varphi = e_8 + \alpha_4 e_7 = \dot{x}_{7d} - \hat{x}_8 + \alpha_4(x_{7d} - \hat{x}_7) \\ s_\theta = e_{10} + \alpha_5 e_9 = \dot{x}_{9d} - \hat{x}_{10} + \alpha_5(x_{9d} - \hat{x}_9) \\ s_\psi = e_{12} + \alpha_6 e_{11} = \dot{x}_{11d} - \hat{x}_{12} + \alpha_6(x_{11d} - \hat{x}_{11}) \end{cases} \qquad (28)$$

Optimizing the sliding mode controller using Genetic Algorithm

Genetic algorithm is defined as a method of solving problems to which no satisfactory, explicit, solution exists.GA is started with a population of strings and thereafter generate successive populations using the following three basic operations: reproduction, crossover, and mutation [17-21].The main feature of GA in this paper is to transform the system output into a cost function in order to find the best values of the control parameters which minimizes the amount of control efforts. This cost function is as follows:

$$J = \sum (x^T R x + u^T Q u) \qquad (29)$$

where R and Q ,square matrixes of twelfth order described in equation (18), areadjusted to provide the most efficient values.

Simulation result

This section accredits the efficiency of proposed model and control scheme by numerical simulation examinations.Table1,summarizes different system parameters of the prototype quadrotor helicopter.

Table 1: Structural parameters

Parameter	Definition	Value	Unit
m	Mass	0.723	kg
l	Arm length	0.314	m
j_r	Rotor inertia	7.32×10^{-5}	kgm^2
I_x	X inertia	8.678×10^{-3}	kgm^2
I_y	Y inertia	8.678×10^{-3}	kgm^2
I_z	Z inertia	3.217×10^{-2}	kgm^2
b	Trust factor	5.324×10^{-5}	N.s^2
d	Drag factor	8.721×10^{-7}	N$m.s^2$
g	gravity	9.81	m/s^2

In addition, full GA-tuned parameters of the designed controller $(\alpha_1,..\alpha_6,,k_1,..,k_6)$ resulted from the computations of section 4.1 have been shown in table 2.

Table 2: Control parameters

Item	Value	Item	Value

α_1	3.58	K_1	8.05
α_2	5.05	K_2	13.65
α_3	3.40	K_3	1.89
α_4	0.60	K_4	0.43
α_5	3.60	K_5	4.26
α_6	1.68	K_6	5.01

Regarding the explanations in section3, the necessaryparameters for simulating the part belonging to estimation of EKBFwill mention as follows. The Number of generated white noise for both ω and ν is considered to 10^6 and the rate of measurement noise power is considered 10^{-4} and the rate of state noise isalso considered 1,both of noises are provided linear which general format of that is as follow:

$$\omega = wgn(10^6, 1, 10^{-4}, 'linear')$$

$$\nu = wgn(10^6, 1, 1, 'linear')$$

according to values of ω and ν , which provided by white noise, R matrix and Q matrix, respectively are considered to be:

$$Q = diag\ [10^{-4} * I_{7\times7}, 10^{-3} * I_{5\times5}] \quad , R = diag\ [10^{-4} * I_{7\times7}, 10^{-3} * I_{5\times5}]$$

also the initial value for P covariance matrix is chosen as $P = diag\ (x^2 + 1)$. The initial conditions for UAV are $x_i(0) = col(0,0,0,0,0,0)$ with $i = 1,..,6$ for translational subsystem and $x_i(0) = col(\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{4})$ with $i = 1,..,6$ for rotational one. Also the desired positions are $x_i(0) = col(2,2,2,2,2,2)$ with $i = 1,..,6$,and desired angles are $x_i(0) = col(0,0,0,0,0,0)$ with $i = 1,..,6$ .The values of the aerodynamic force coefficient $[C_x, C_y, C_z]$ are set to 0.25, and stop when angles ϕ and θ and ψ are stabilized to zero values.

Fig.2 shows the response of the nonlinear controller to stabilize the quadrotor during hovering. It can be seen that the controller succeeded in controlling the roll, pitch, and yaw angles of the quadrotor in less than 2s. Fig.3 shows quadrotor dynamics are stabilized following the given position. Fig.4 and Fig.5, show the estimated errors results, obtained for the attitude and position stabilization of the mini aircraft controller based EKF observer that ensures a good tracking presents. Fig.6 shows the stability rotor speed response of a quadrotor during hovering. Finally,Fig.7 indicates an excellent reference tracking performance even if external disturbances originated by aerodynamic forces and moments are considered.

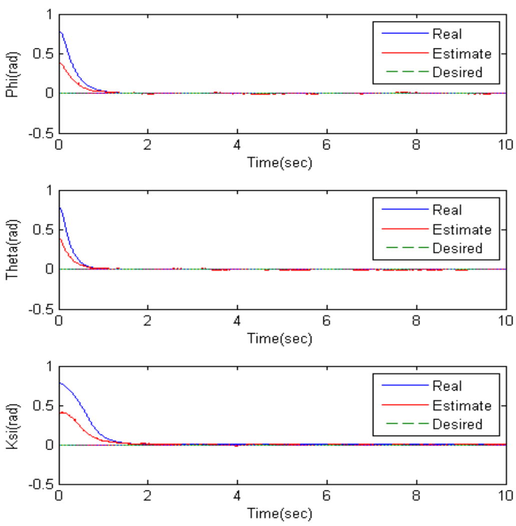

Figure2.Tracking results forthe orientation (ϕ, θ, ψ)

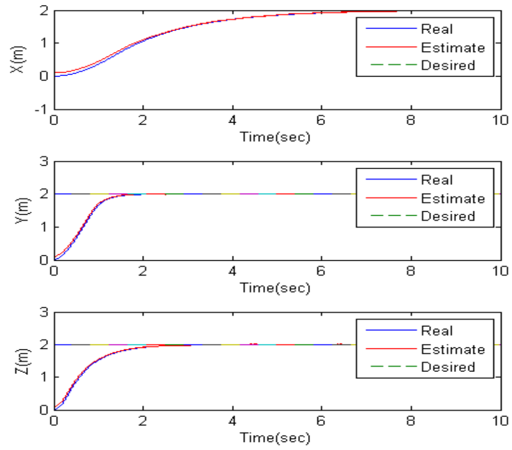

Figure 3.Tracking results for theposition (x, y, z)

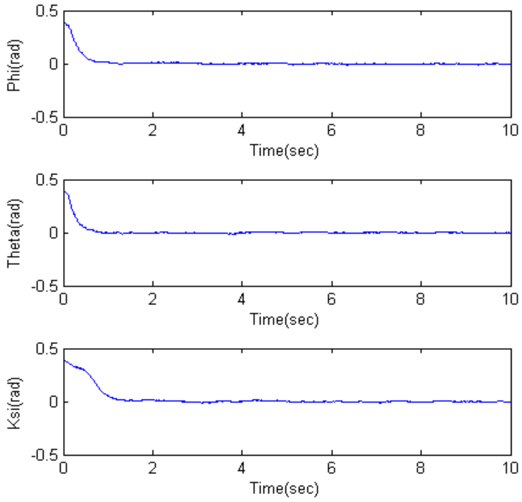

Figure 4.Estimation errors due to trajectories of (ϕ, θ, ψ)

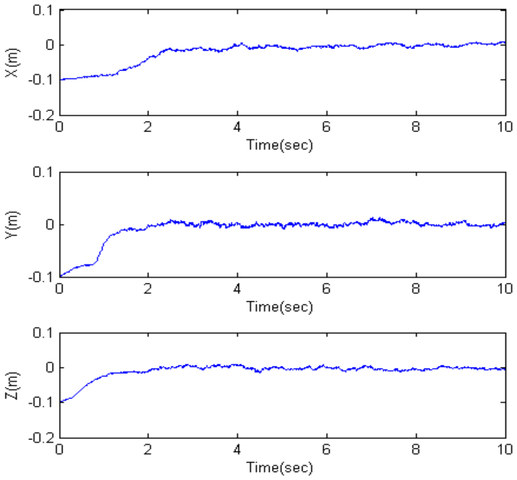

Figure 5.Estimation errors due to trajectories of(x, y, z)

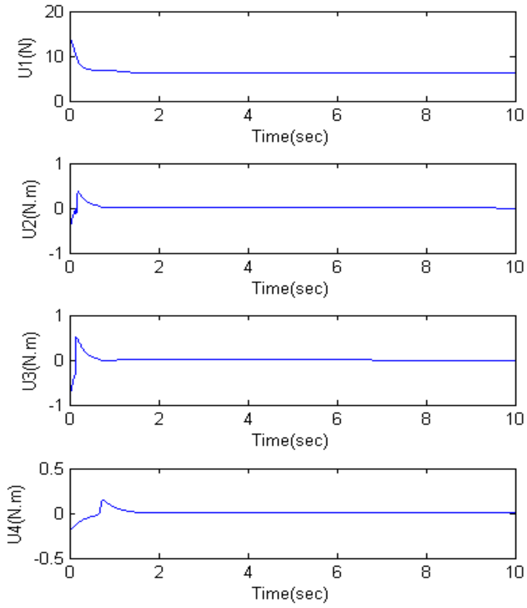

Figure 6. Control inputs of thequadrotor helicopter

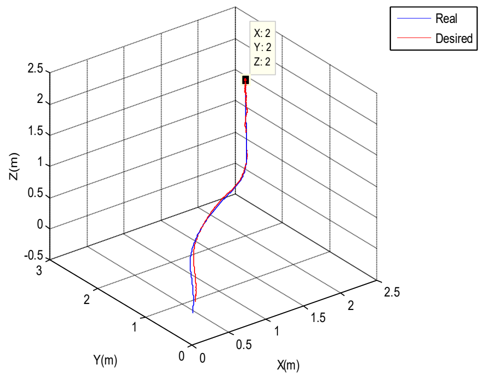

Figure 7.Global trajectory of the quadrotor

CONCLUSION

In this paper, a novel stabilizing control laws synthesis were presented based on sliding mode technique along with optimal tuning of control parameters by using genetic algorithm. In modeling of the quadcopter system, we considered the gyroscopic effects and generated the dynamic equations with Newton-Euler method. The developed control laws allowed the tracking of various desired trajectories expressed for the center of mass coordinates of the system. A nonlinear observer (EKBF) was also introduced to estimate unmeasured states of the system.The ability of the nonlinear controller in stabilizing the quadrotor system is examined through various simulations and the results indicate effectiveness of the proposed control strategy.

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