



## **Numerical Analysis of Heat Transfer in the Unsteady Flow of a non-Newtonian Fluid over a Rotating cylinder**

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### **ABSTRACT**

*In this article, forced convection heat transfer over a rotating cylinder with fixed wall temperature is examined and studied in the flow of a non-Newtonian fluid in the two-dimensional and unsteady condition. Governing equations of flow are continuity, momentum, and energy which have been solved by numerical finite volume method, assuming constancy of thermo-physical properties of fluid. This solution is performed on the basis of fixed values of Reynolds number = 100, and Prandtl number = 1, and in the power-law index range from 0.4 to 1.8, and Rossby numbers = 0, 1, 2, and 4, and the effects of power-law index and Rossby number on the value of local and average Nusselt number are discussed and examined.*

**Key words:** Rotating cylinder, non-Newtonian fluid, unsteady flow, convective heat transfer, numerical analysis

### **INTRODUCTION**

The flow pass a cylinder, represents a classical problem in fluid mechanics, and acceptable results have been produced during last 50 years. This flow represents the purpose of many industries such as: the flow in pipe and pin fin heat exchangers, in using thin wire as a measuring tool, in thermal processing of the particles of foodstuffs etc. Engineering science, in recent years, has dealt with various aspects of this subject such as flow regimes and their transfer, parameters like hydrodynamic forces (lift coefficient, and drag coefficient) exerted on cylinder immersed in the fluid flow, heat degree and mass transfer and the detailed structure of the flow field, like: wake phenomenon, vortex separation of isotherm patterns etc. Very good articles and even books that have exclusively addressed the Newtonian fluid over the cylinder are now available [1-3]. On the other hand, the industries encounter materials with multi-phase structure or materials with high molecular weight (suspension of paper and pulp, foodstuffs, polymers, foam etc.) which show stress-related behavior [4]. Obviously, many of the non-Newtonian fluids, especially polymeric systems show viscoelastic behavior. Few studies are available for the creeping flow past a cylinder that show the viscoelastic effects of this flow configuration, in addition, these studies have examined the flow rarefaction effect very little. On the other hand, in few experiential studies, experimented fluids simultaneously show stress-related and viscoelastic behaviors. As far as investigated, no studies have been done on convective heat transfer from a heated rotating cylinder to non-Newtonian fluid with unsteady flow, so this subject has been discussed in this article. Here, studies which addressed Newtonian fluid flow in unsteady flow regime and/or non-Newtonian fluids past a cylinder, are indicated briefly and in a limited way, and next we will discuss and examine new results.

A study that examines the oscillatory rotation effect of the cylinder on heat transfer. In this article, time-related average values of drag coefficient and heat transfer for two different fluids, air and water in  $Re = 200$ , have been reported [5], a numerical study which has been particularly performed with regard to heat transfer from a cylinder to the air in Reynolds number range ( $Re < 200$ ), tries to prove the effects of temperature-related properties on heat transfer [6], conversely, not only have studies concerning the flow of non-Newtonian liquids over the cylinder been done recently, but they have been also limited to the steady flow [7-11]. Similarly, the mixture convection role has been cleared recently. In cross-flow and as a

help to buoyancy configuration and under the right conditions, free convection heat transfer can generally boost heat transfer rate of the shear thinning fluid ( $n < 1$ ) 40-45% [12-13], therefore, there is right amount of information about convective heat transfer of a cylinder in non-Newtonian fluids. Assuming steady flow, these results are limited to a maximum value of Reynolds number ( $Re = 40$ ). In a study which has been performed about forced convection heat transfer of immersed cylinder in non-Newtonian flow with unsteady regime, in wide range of Reynolds number ( $Re = 40 \sim 140$ ), Prandtl number ( $Pr = 1 \sim 100$ ), and power-law index ( $n = 0.4 \sim 1.8$ ), the heat transfer parameters have been compared [14].

The present work has examined the effect of the cylinder rotation on the value of forced convection heat transfer parameter, in non-Newtonian unsteady flow with the fixed value of Reynolds number of ( $Re = 100$ ) and Prandtl number of ( $Pr = 1$ ) for power-law index range of ( $n = 0.4 \sim 1.8$ ). The effect of power-law index and Rossby number on the value of local and average Nusselt number is discussed and examined by solving the governing equations of flow, including: continuity, momentum, and energy, using the numerical method.

### Defining the problem and governing equations

The flow is two-dimensional, unsteady, incompressible, with a uniform velocity of  $U_0$  and fixed temperature of  $T_0$  over an infinitely long cylinder, the cylinder surface temperature is assumed to be fixed ( $T_0 < T_w$ ). Thermo-physical properties of the fluid ( $c_p, \mu, k, \rho$  etc) is independent of temperature, and the loss due to viscosity can be neglected. These two assumptions cause to solve the momentum and energy equations separately.

$$\Delta T = T_w - T_0 \quad (1)$$

As it can be observed in figure (1), the flow is simulated in a channel which its walls are assumed far enough from the cylinder, so that the effects of the boundaries on the flow over the cylinder are minimized, also the boundary conditions, on the top and bottom walls, are assumed to be sliding.

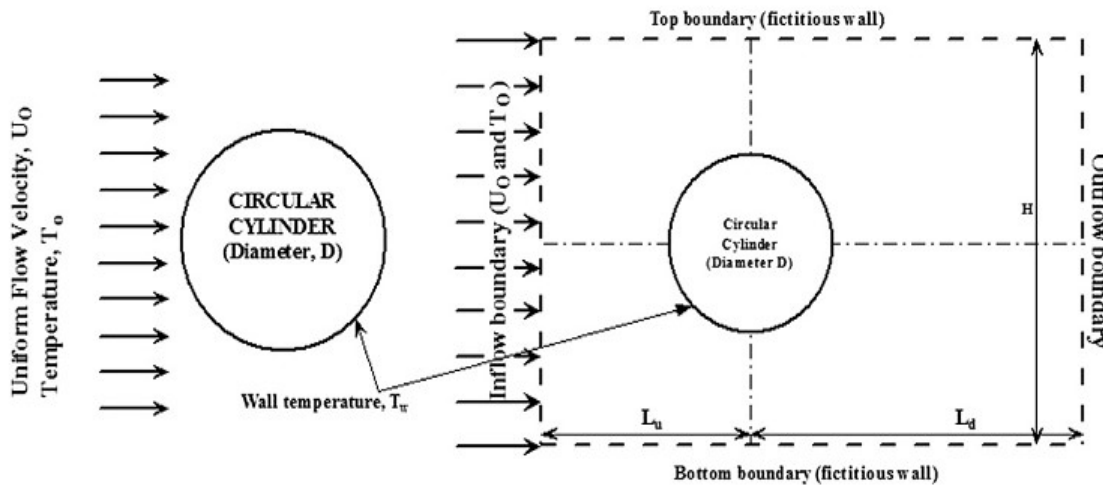


Figure 1: The geometry of the problem

The flow and temperature are analyzed by continuity, momentum, and energy equations:

$$\nabla \cdot U = 0 \quad (2)$$

$$\rho \left( \frac{\partial U}{\partial t} + U \cdot \nabla U - f \right) - \nabla \cdot \sigma = 0 \quad (3)$$

$$\rho c_p \left( \frac{\partial T}{\partial t} + U \cdot \nabla T \right) - K \nabla^2 T = 0 \quad (4)$$

Boundary conditions:

In the input boundary: The uniform flow is in x direction and the input liquid temperature degree is uniform

$$U_x = U_0, \quad U_y = 0, \quad T = T_0 \quad (5)$$

The surface of the cylinder: No-slip boundary condition and fixed temperature

$$U_x = 0, U_y = 0, T = T_w \quad (6)$$

The output boundary: The atmospheric pressure condition is used.

In the top and bottom boundary: Since these walls are assumed walls, sliding flow and adiabatic temperature conditions are exerted.

$$\frac{\partial U_x}{\partial y} = 0, U_y = 0, \frac{\partial T}{\partial y} = 0 \quad (7)$$

In this report, the flow and heat transfer are examined on non-Newtonian fluids which their behaviors are defined by power-law rules. The dimensionless parameters of Reynolds, Prandtl, and Nusselt are defined for these fluids as follows:

$$Re = \frac{\rho D^n U_0^{2-n}}{m}, \quad Pr = \frac{m c_p}{k} \left( \frac{U_0}{D} \right)^{n-1} \quad (8)$$

As it can be observed, unlike Newtonian liquids, Prandtl number for the fluids which follow the power-law rule, is dependent on the cylinder velocity and diameter, in addition to thermo-physical properties.

$$Nu(\theta) = \frac{hD}{k} = -\frac{\partial T}{\partial n_s}, \quad n_s = n_x e_x + n_y e_y \quad (9)$$

$$Nu = \frac{1}{2\pi} \int_0^{2\pi} Nu(\theta) d\theta \quad (10)$$

This is Nusselt number average value that is often required in engineering design calculation process in order to estimate heat transfer from or to the isothermal cylinder, or to estimate the temperature of the cylinder for a heat transfer rate.

And finally another dimensionless number which enters the equations, due to the cylinder rotation, is Rossby number.

$$Ro = \frac{\omega L}{U_0} \quad (11)$$

### Validation

To make sure about the correctness of numerical results, the results produced from Reynolds number of 120 and Prandtl number of 0.7 over the surface of fixed temperature cylinder are compared with Vijaya et al results [14] in figure (2). An acceptable accordance can be observed between two cases.

The network which has been intended for this problem, is created unsystematically and the cells are triangular. The triangular cells on the surface of the circular cylinder provide a better coverage. In order to evaluate the independence of the results from the network, the results obtained from a network of 151078 cells were compared with a network of 207724 cells, which fully correspond.

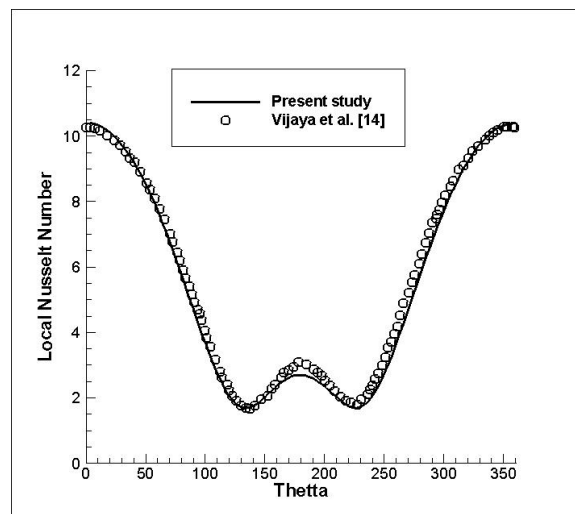


Figure 2: Validation of the intended model

## RESULTS AND DISCUSSION

Figures 3, 4, and 5 show the diagrams of Nusselt number for Reynolds number of 100 and Prandtl number of 1 and for power-law indices of 0.4, 1, and 1.8 over the surface of the cylinder for Rossby numbers of 0, 1, 2, and 4.

As it has been shown, Nusselt numbers range of the flow has partly become narrower from Newtonian fluid ( $n = 1$ ) towards the thickening fluid ( $n > 1$ ), while it unlikely has a wider range from Newtonian fluid ( $n = 1$ ) towards thinning fluid ( $n < 1$ ), that these differences are more tangible in the forehead part of the cylinder.

The high degree of the effect of power-law index on heat transfer rate is completely determined and cleared by comparing amongst local Nusselt numbers range and therefore average Nusselt number of thickening fluid ( $n > 1$ ) and thinning fluid ( $n < 1$ ).

It also can be stated that for rotating cylinders, the average value of Nusselt number over the surface of the cylinder, for thickening fluid ( $n > 1$ ) and Newtonian fluid ( $n = 1$ ), decreases as Rossby number increases, that the rate of this decrease is greater in thickening fluids, but in thinning fluids ( $n < 1$ ), this procedure is completely opposite, and the average Nusselt number moves in an increasing slope. This point can obviously be seen in the diagram of figure 6.

The boundary layers formed in thinning fluids are much narrower than the ones in thickening fluids. Therefore, the cylinder surface can has a better heat exchange with the environment, but in thickening fluids, as viscosity increases the boundary layer thickness increases, and this issue would prevent from the proper heat transfer between the cylinder surface and the environment. Also the cylinder rotation decreases the viscosity of thinning fluids and the thickness of the boundary layer, but in thickening fluids the opposite happens.

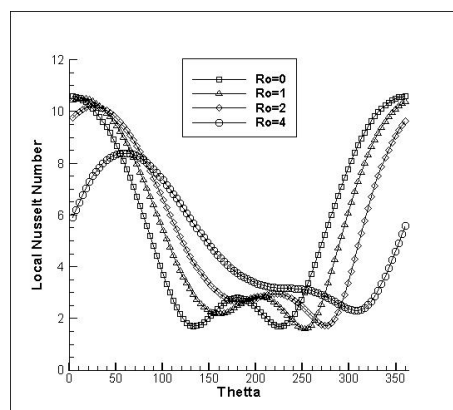


Figure 3: Local Nusselt number over the cylinder surface, for  $n = 1$

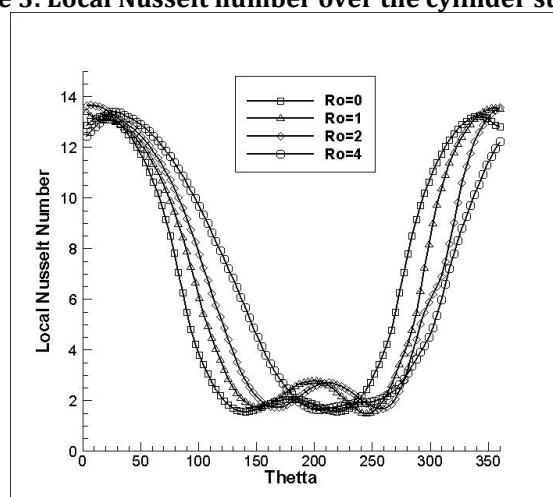


Figure 4: Local Nusselt number over the cylinder surface, for  $n = 0.4$

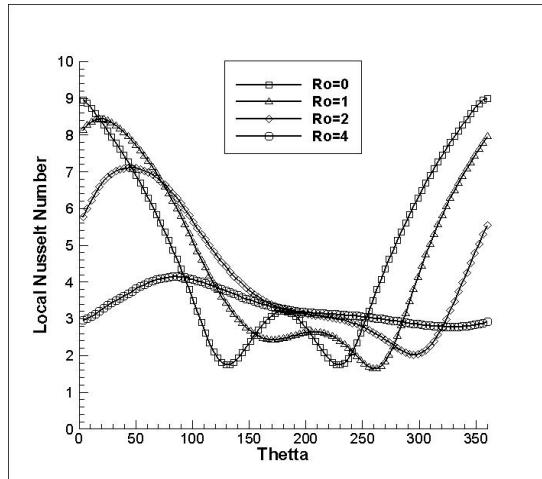


Figure 5: Local Nusselt number over the cylinder surface for  $n = 1.8$

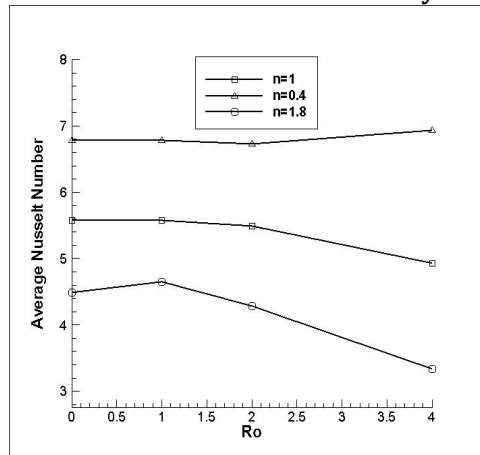


Figure 6: Average Nusselt number over the cylinder surface, based on Rossby number and for power-law indices of 1, 0.4, and 1.8

## CONCLUSION

The numerical results show that it can be generally concluded, that in the same flow conditions, the higher heat transfer rate can be obtained in thinning fluids ( $n < 1$ ), in relation to Newtonian fluids and thickening fluids ( $n > 1$ ).

Also in order to improve heat transfer in rotating cylinders, the use of thinning fluids has advantages in relation to Newtonian and thickening fluids, and in this kind of fluids, an increase in the rotation speed leads to an increase in heat transfer.

## List of symbols

Re	Reynolds number
Pr	Prandtl number
$Nu$	Nusselt number
$Ro$	Rossby number
$m$	Compatibility coefficient
$n$	Power-law index
$D$	Hydrodynamic diameter, $m$
$\rho$	Density, $kg/m^3$
$\mu$	Viscosity, $Pa.s$
$\omega$	Angular velocity, $rad/s$
$U_0$	Free flow velocity, $m/s$
$L$	Property length, $m$

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