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## **Analysis of square hole drilling mechanism using Oldham's non-coaxial couplings**

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### **ABSTRACT**

There are several methods to creating square hole such as drilling with a CNC machine, electrical discharging and broaching. These methods have complexity of manufacture along with high cost. Creating square hole using drill, in addition to has an easier manufacturing process, also is more affordable. In this paper, curves of constant width were discussed first of all and then, a mathematical model was achieved for the shape of the cam of constant width by using the properties of these curves. Cam of constant width can truly rotate inside of a square frame. So, by designing a bit on cam it would be creating square hole on the work piece. During the rotation of the cam, the center of rotation is not fixed and this is the main problem in this mechanism. So there is a need to a non-coaxial coupling to transmit the rotational motion. In this paper Oldham non-coaxial coupling has been examined and results were shown that the rotational velocity of input and output shafts are equals. So this coupling can be used for transmitting the rotational motion from drill to the cam and bit.

**KEY WORDS:** curves of constant width, square hole, Oldham non-coaxial coupling, Reuleaux polygons, CNC machining method, broaching machining method.

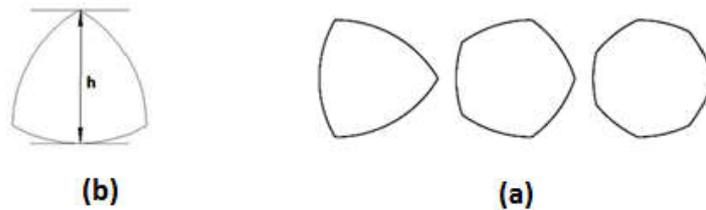
### **INTRODUCTION**

Making square holes have a widely usage in industry, such as creating square hole on a shaft and coupling that with a square bar. There are different ways to create a square hole. One of these methods is creating the square hole with CNC machine. It is so accurate by using CNC machine but it is so costly. In this method, the plan of hole design by CAD software, and then, after becoming to G-Code, it transferred to CNC machine [1].

Electrical Discharge Machining is another method to create a square hole on a workpiece. EDM is a process that uses electrical discharges from an electrode to workpiece, regardless of the mechanical properties such as strength and hardness [2]. In this machining method, the electrode is constructed to the shape of desired hole and by creating electrical discharge between tool electrode and work piece, the spark is generated and the shape of electrode on work piece will be created [3]. There is no contact between tool electrode and work piece in this method.

Broaching machining method is another method that can be used to create a square hole on the surface of a workpiece. Broaching is commonly used for machining of internal profiles or external complex profiles which are difficult to generate by other machining methods. In fact, broaching method was developed for non-circular internal profiles, such as square hole [4].

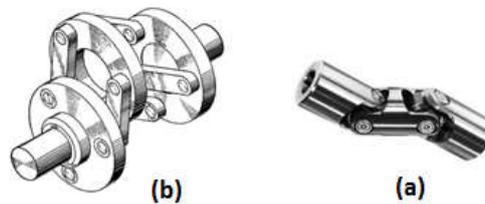
Square hole drilling mechanism, including a triangle cam, that can have rotational motion within a square frame. Curves of constant width are proposed to analysis the mathematical model of the cam. Curves of constant width provide a great example of vector space and also make it possible to have access to a large amount of information. Particularly, Reuleaux polygon's, are a special set of curves of constant width [5]. Reuleaux polygons are formed by connections through non-uniform circular arcs to each other [6] (Figure 1.a). The curves of constant width have a plane shape which has constant width in all directions. The mentioned term, width, is a vertical distance between two parallel lines that are tangent to curve (Figure 1.b).



**Figure 1 : (a) curves of constant width, (b) the width of CCW in vertical direction**

Because of the constant width of the Reuleaux polygon in all directions, it can rotate between two parallel lines. Therefore it is capable to rotate in a square, which is actually included of couple of two parallel lines. So, if a cam with Reuleaux shape rotates a cycle in a square frame, it will covers the whole square surface and creating a square hole.

The most important point is that, during the rotation of the Reuleaux polygon, the center of rotation will not stay at a fixed point. So, in order to transmit the torque from drill to the cam, it is needed to use a non-coaxial coupling which can transmit torque between non-aligned axes. Oldham coupling, Universal coupling or Hook coupling (figure 2.a) and Richard Schmidt coupling (Figure 2.b) are some examples of non-coaxial couplings.



**Figure 2 : (a) Universal coupling, (b) Richard Schmidt Coupling**

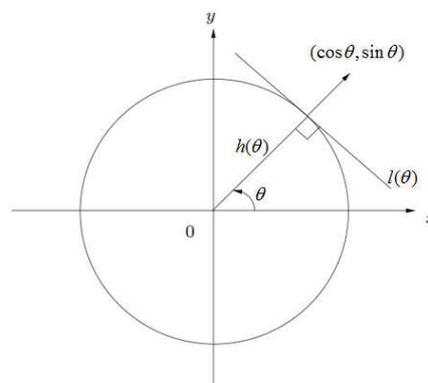
The inventor of the coupling, Richard Schmidt of Madison, Alabama said that a similar link arrangement had been known to some German engineers for year. But those engineers were discouraged from applying the theory because they erroneously assumed that the center of disk had to be retained by its own bearing. Actually Schmidt found that the center disk is free to assume its own center of rotation. In operation, all three disks rotate with equal velocity [7].

Hosseini et al. [8] investigated the equality of input and output rotational velocity in the Universal coupling and usage of this coupling in square hole drilling.

In this paper, the mechanism of square hole drilling using Oldham coupling has been studied. A mathematical model used for cam mechanism is provided in the first part and then, Oldham coupling for transmitting the torque from drill to the cam and the bit were studied. Finally has been presented a model of the drill including the square hole mechanism.

### Mathematical Analysis of square hole drilling mechanism

To find the equation of the cam's curve, we will use the most important feature of the curves of constant width, which is the constant width in all direction. The simplest curve of constant width is circle. So we will consider it as a base of the calculations to find the equation of cam's curve and then, envelope the results to all kinds of curves of constant width.



**Figure 3 : The simplest curve of constant width (circle)**

According to figure 3, curve's width has been measured:

$$h(\theta) + h(\theta + \pi) = k \quad (1)$$

In this equation,  $h$  shows the line where it is perpendicular to the drawn tangent.  $\theta$  shows the angle where  $h$  makes with the horizontal line.  $k$  is a constant. The collection of the tangent to the curve is [9]:

$$F(x, y, \theta) = x \cdot \cos \theta + y \cdot \sin \theta - h(\theta) \quad (2)$$

The expansion of the tangent to the curve's equation could be calculated by finding the derivative of equation 2 in term of  $\theta$ . So a system of equation will be formed like this.

$$\begin{aligned} F &= 0 \\ \frac{\partial F}{\partial \theta} &= 0 \end{aligned} \quad (3)$$

$x$  and  $y$  variables will be found by solving the system of equation 3:

$$\begin{aligned} x(\theta) &= h(\theta) \cdot \cos \theta - h'(\theta) \cdot \sin \theta \\ y(\theta) &= h(\theta) \cdot \sin \theta + h'(\theta) \cdot \cos \theta \end{aligned} \quad (4)$$

Variety curves of constant width equations are obtained by choosing  $h(\theta)$  to different forms. It should be noted that  $h(\theta)$  should satisfy equation 1. For example, by choosing  $h(\theta) = 1$ :

$$\begin{aligned} x(\theta) &= \cos(\theta) \\ y(\theta) &= \sin(\theta) \end{aligned} \quad (5)$$

This is the equation of the circle by unit radius.

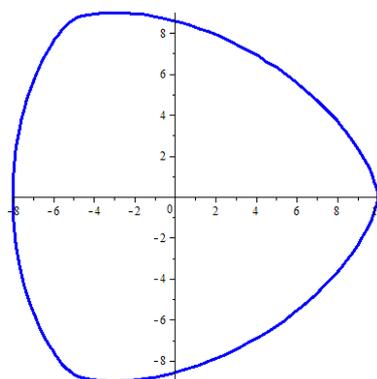
By considering  $h(\theta) = 2 \cos^2(\frac{3\theta}{2}) + 8$ ,  $x$  and  $y$  variables will be [9, 10]:

$$\begin{aligned} x(\theta) &= (\cos 3\theta + 9) \cdot \cos \theta + 3 \sin 3\theta \cdot \sin \theta \\ y(\theta) &= (\cos 3\theta + 9) \cdot \sin \theta - 3 \sin 3\theta \cdot \cos \theta \end{aligned} \quad (6)$$

Equation (6) shows a curve of constant width. Now by choosing  $\theta = 0$  and replacing  $h$  in equation 1, the value of  $k$ , which is the curve's width, will be achieved.

$$k = h(0) + h(\pi) = [2 \cos^2(0) + 8] + [2 \cos^2(\frac{3\pi}{2}) + 8] = 18 \quad (7)$$

The diagram of equation (6) has been shown in figure (4). It is obvious from figure that the cam width is 18.

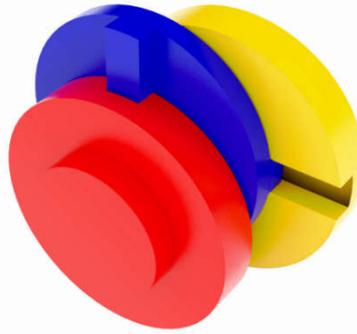


**Figure 4 : Curve of constant width presented in Eq.(6)**

Figure (4) is showing a curve of constant width that can have rotational motion between two horizontal parallel lines with distance of 18. It also can have rotational motion between two vertical parallel lines. So, a cam with this shape has rotational motion in a square by 18 side length.

#### **Oldham coupling for transmitting the torque**

The center of rotation in square hole mechanism is not fixed. So it is impossible to transmit the rotational motion from drill to the cam with coaxial couplings and it is needed to use a non-coaxial coupling. Oldham coupling can be used for this operation. This coupling was invented by John Oldham in 1821 in Ireland and first has been used in rowboats for paddling. This coupling is including three disks (figure 5). The first disk and the third disk are connected to the input and output axels respectively. The second disk is in contact with first and third disks with groove.

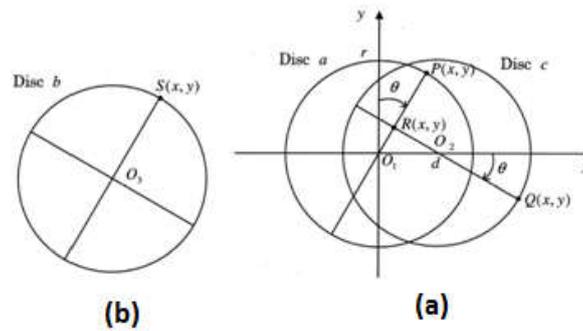


**Figure 5 : Oldham coupling**

Velocity of the input and output axels must be equal in this mechanism. Otherwise, the bit would not rotate with a constant velocity and therefore the drilling will be Inappropriate. In the following rotational velocity of input and output of the coupling has been studied.

Input disk (a) and output disk (c) of the coupling has been shown in figure 6.a. The middle disk has been shown in figure 6.b.  $P$  is at end of the input disk's groove.  $Q$  is at end of the output disk's groove. These points have a constant circular motion with radius of  $r$  around  $O_1$  and  $O_2$  respectively. It should be noted that the distance between  $O_1$  and  $O_2$  has been shown with  $d$ .  $R$  is the crossing point of two grooves. This point is the center of middle disk,  $O_2$ , and it has a circular motion about coordinate  $(\frac{d}{2}, 0)$  and radius  $\frac{d}{2}$ .

Also  $S$  is at end of the middle disk's groove.



**Figure 6 : (a) Input and Output disks, (b) middle disk**

The coordinates of  $P$ ,  $Q$ ,  $R$  and  $S$  are:

$$\begin{aligned}
 P &= [r \cos(\frac{\pi}{2} - \theta), r \sin(\frac{\pi}{2} - \theta)] \\
 Q &= [r \cos(-\theta) + d, r \sin(-\theta)] \\
 R &= [\frac{d}{2}(1 - \cos 2\theta), \frac{d}{2} \sin 2\theta]
 \end{aligned} \tag{8}$$

$$S = P + R = [r \cos(\frac{\pi}{2} - \theta) + \frac{d}{2}(1 - \cos 2\theta), r \sin(\frac{\pi}{2} - \theta) + \frac{d}{2} \sin 2\theta]$$

The components of velocity of these points will be derived by differentiation (8) in term of time.

$$\begin{aligned}
 \frac{dP}{dt} &= [r \omega \sin(\frac{\pi}{2} - \theta), -r \omega \cos(\frac{\pi}{2} - \theta)] \\
 \frac{dQ}{dt} &= [r \omega \sin(-\theta), -r \omega \cos(-\theta)] \\
 \frac{dR}{dt} &= [d \omega \sin 2\theta, d \omega \cos 2\theta]
 \end{aligned} \tag{9}$$

$$\frac{dS}{dt} = [r \omega \sin(\frac{\pi}{2} - \theta) + d \omega \sin 2\theta, -r \omega \cos(\frac{\pi}{2} - \theta) + d \omega \cos 2\theta]$$

In above equations the first and second components shows the velocity in x and y directions respectively.

$$V = \sqrt{V_x^2 + V_y^2} \rightarrow \begin{cases} V_P = r \omega \\ V_Q = r \omega \\ V_R = d \omega \end{cases} \quad (10)$$

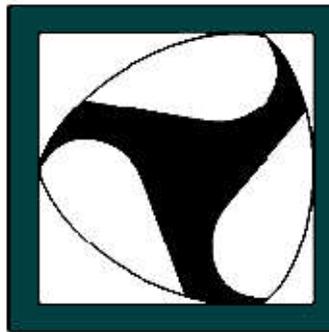
According to the mentioned equations, the velocity of input disk and output disk is equal together. The velocity of second disk is depends on d and calculate from equation (11).

$$V_S = \sqrt{(r \omega \sin(\frac{\pi}{2} - \theta) + d \omega \sin 2\theta)^2 + (-r \omega \cos(\frac{\pi}{2} - \theta) + d \omega \cos 2\theta)^2} \quad (11)$$

The velocity of first disk and third disk are not depends on the second disk. The velocity of the second disk takes different values for different angels. The suitable coupling for the square hole mechanism is the one that has equal velocity for input and output axels. So, Oldham coupling is suitable for this mechanism.

**Conclusion**

According to the calculations, a curve of constant width can have rotational motion in a square. So a cam with the shape of the curve of constant width can rotate in a square frame and therefore, it would be creating square hole on the work piece by designing a bit on the cam. Bit, cam and square frame has been shown in figure 7.

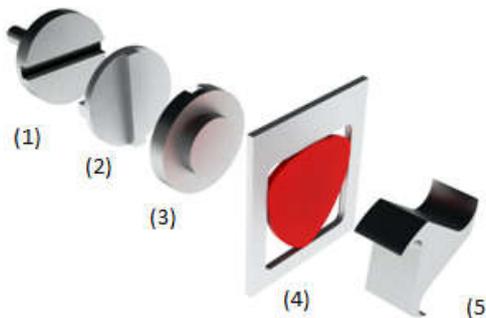


**Figure 7 : Bit, CCW cam and square frame in front view**

It is needed to use a non-coaxial coupling in order to transmit the rotational motion from drill to the cam. Oldham coupling is suitable for this purpose because of the equal velocity of the input and output axels of it.

In figure (8), it has been shown that how Oldham coupling, cam and bit are joined together. According to the figure (8), number 1, 2, 3, 4 and 5 shows input disk, middle disk, output disk, cam of constant width and the bit respectively.

In deferent views, it has been shown that how bit, cam of constant width and Oldham coupling joined together.



**Figure 8 : The connection of Oldham coupling, CCW cam and bit**



**Figure 9 : Square hole drilling mechanism using Oldham coupling from different view**

#### REFERENCES

1. Alan Overby, 1986. *Cnc machining handbook*. McGraw-Hill
2. Ben Fleming, 2005. *The EDM How To Book*. Fleming publications
3. K.Y. Song, D.K. Chung, M.S. Park, C.N. Chu, 2013. *Development of strip EDM*. Procedia CIRP, 6, Jun, pp. 53-57
4. U. Kokturk, E. Budak, 2004. *Optimization of broaching tool design*. intelligent Computation in Manufacturing Engineering – 4, Faculty of Engineering and Natural Sciences, Sabanci University, Orhanli, Tuzla, 34956, Istanbul, Turkey
5. Peter Fumich, Candice Quinn, David Freelan, Paul Sharaba, Josh Orzel, 1987. *Curves of Constant Width and Reuleaux Polygons*. Mathematics magazine, 60(3), Jun, pp.131-140
6. Kupitz, Y.S., Martini, H., 2000. *On the isoperimetric inequalities for Reuleaux polygons*. J. Geom., 68, pp. 171-191
7. Neil Sclater, Nicholas P. Chironis, 2007. *Mechanisms and Mechanical Devices Sourcebook*. McGraw-Hill
8. Shahram Hosseini Chaleshtori, Ehsan Zamani, Ahmadreza Ashrafi, 2014. *Square hole created by drilling down by using Universal non-coaxial couplings*. Indian J.Sci.Res.1(2), pp. 583-586
9. Andrew David Irving, 2006. *Curves of Constant Width & Centre Symmetry Sets*. MSc mini-dissertation, Jun, pp. 1-45
10. Yutaka Nishiyama, 2009. *From Oldhams Coupling to Air Conditioners*, Osaka Keidai Ronshu, September, 60(3), pp. 77-84