Designing, Modeling, and Simulating a Single-phase Active Power Filter for Harmonic Compensation and Reactive Power

Ali Asgari Asl1,2, Aydin Sakhavati3
1-Department of Power Engineering, Tabriz Science and Research Branch, Islamic Azad University, Tabriz, Iran
2-Department of Power Engineering, Tabriz Branch, Islamic Azad University, Tabriz, Iran
3-Department of Power Engineering, Tabriz Branch, Islamic Azad University, Tabriz, Iran

ABSTRACT
In today's distribution networks, with extensive application of non-linear and sensitive loads including electronic power transformers and sensitive electronic devices, problem of power quality is increasingly considered, and these loads create harmonic pollution by taking non-sinusoidal currents from the network. Since load-resulted power quality disturbances affect the source, and voltage disturbances affects good service to the subscribers, using reformers seems inevitable. The most significant power quality disturbance that subscribers apply to the network is harmonic pollution. Current harmonics, due to voltage drop on the network elements cause undesired operation of sensitive electrical equipments and protection devices. Also the overvoltage caused by the occurrence of parallel resonance between inductance of power network and reactive power compensation capacitor bank damage the network equipments such as power factor correction capacitors. This paper presents a method for improving power quality using parallel active filters with the application of vector control method, and studies this filter function in compensating and removing harmonic distortion, correcting power factor, unbalanced load conditions, and changes in load conditions.

Keywords: power quality, current harmonic, reactive power, active filter, harmonic compensation

INTRODUCTION
Manufacturers of electric equipments define power quality as the proper functioning of devices based on the specifications of power supplies. The way electrical equipments are energized in a manner that is appropriate for their performance and adjacent equipments is called proper power quality. Thus, any deviation from the ideal conditions of the power network resulted in malfunctions of the system is called power quality problem.

Technically, in engine loop terms, power is the rate of energy transfer and is proportional to the product of voltage and current. The significant qualitative definition of this quantity is difficult. The distribution network can only control voltage quality and has no control over the current of a specific load. Therefore, power quality standards can only specify the allowed limits of the source voltage [1].

Total harmonic distortion and effective value
There are several common measures to show the size of a wave harmonic by a number. One of the most common ones is the total harmonic distortion (THD) that can be obtained for voltage or current.

\[ \text{THD} = \sqrt{\sum_{h=2}^{\infty} \left( \frac{M_h}{M_1} \right)^2} \]

Where, \( M_h \) is effective value of the \( h^{th} \) harmonic component of the quantity \( M \). THD is effective value of harmonic components of a distorted wave, and represents heat energy of harmonics compared to the original value.

Devices to improve power quality
One of devices which can be used to improve the power quality are filters. Filters can be used to solve problems such as unbalance, harmonics, and reactive power in power networks, and active and passive filters are the most important ones [2].
Parallel passive filters
Parallel passive filters are the most common tools for restricting harmonic in the industry. This type of filters that are in parallel with the distorting load, have a high impedance in the main wave frequency and have low impedance at a certain harmonic frequency that are scheduled to be filtered [3].

Parallel active filters
Parallel active filters compensate harmonic currents generated by the load by injecting a harmonic current in the same size but opposite sign. In these circumstances, the active filter acts as a current source where the size of current generated is equal to the size of the harmonic components of the load current, and its phase is shifted 180° [4].

Series active filters
These filters inject a harmonic voltage in series way with a voltage source and act as a controlled voltage source which is capable of compensating voltage source harmonics and its unbalance [5].

ACTIVE FILTER MODELING
Various methods for the control of active filters have been presented since 1971. In general, these methods can be divided on based on how to extract signal harmonics to methods in the frequency and time domains [6]. Figure 1 shows the model of the single-phase active power filter, where \( i_L(t) \) is the nonlinear load current, \( i_F(t) \) is the active filter, \( i_S(t) \) is the network current, \( L \) is the size of coupling filter between the filter and the network, and \( C \) is DC capacitance link.

![Figure 1. Model of the active filter](image1.png)

The circuits equal to proposed active filter when keys \( S_1, S_4, S_2 \), and \( S_3 \) are connected are shown in the following figures, respectively.

![Figure 2. Circuit of the proposed active filter with the keys \( S_1 \) and \( S_4 \) connected](image2.png)
According to the circuit in Figures (2) and (3), we have the following equations respectively:

\[ L \frac{di_{LF}(t)}{dt} = -v_c(t) - v_s(t) \]  
\[ (2) \]

\[ C \frac{dv_c(t)}{dt} = -i_{LF}(t) \]  
\[ (3) \]

\[ L \frac{di_{LF}(t)}{dt} = v_c(t) - v_s(t) \]  
\[ (4) \]

By taking the average of equations (2) and (4):

\[ L \frac{di_{LF}(t)}{dt} = (2d(t) - 1)v_c(t) - v_s(t) \]  
\[ (5) \]

Controller design

Our controller consists of 2 inner loops (current loops) and outer loops (voltage loops). First, we start with designing the controller. Assuming that DC link voltage is stabilized in its reference, equation (5) with a good approximation can be considered as the following:

\[ L \frac{di_{LF}(t)}{dt} = 2v_c d(t) - v_c(t) - v_s(t) \]  
\[ (7) \]

Thereby, the control model required to design the inner loop (current loop) controller can be implemented based on the equation (7):

\[ d(t) = \frac{1}{2v_c} \left[ L \frac{di_{LF}(t)}{dt} + v_c(t) + v_s(t) \right] \]  
\[ (8) \]

The current loop model based on equation (8) is shown in Figure 4. \( i_{ref}(t) \) is the reference current that should be tracked by \( i_{LF} \) and generated by the outer loop (voltage loop).
Figure 4. The inner loop (current loop) controller

To eliminate the effects of network voltage and DC link voltage, a feedback path is added to the controller, so that current $i_{ref}$ can follow the reference value of $i_{ref}(t)$ with a reasonable accuracy. The conversion function of the controller is as follows:

$$G_{p2}(S) = \frac{K_{p2}S + K_{I2}}{S}$$  \hspace{1cm} (9)

Where $k_{p2}$ and $k_{I2}$ are constants. The conversion function of inductor current into the reference current according to figure (4) is obtained as follows:

$$I_{f}(S) = \frac{2V_{c}(K_{p2}S + K_{I2}S)}{LS^2 + 2K_{p2}V_{c}S + 2K_{I2}V_{c}} I_{r}(S)$$  \hspace{1cm} (10)

As we know, the current of active filter includes nonlinear load harmonics. Therefore, for the harmonic current to be able to track its reference current, the control loop bandwidth must be large enough to cover the harmonics of desired number. The equation of the conversion function for equation (10) is as follows:

$$\Delta_{I}(s) = LS^2 + 2K_{p2}V_{c}S + 2K_{I2}V_{c}$$  \hspace{1cm} (11)

Comparing equation 11 with the classic form of conversion function, that is $\Delta(s)=S^2+2\xi\omega_{n}S+\omega_{n}^2$, the following equations are obtained for the natural frequency of system and the resulted damping rate:

$$\omega_{n} = \sqrt{\frac{2K_{I2}V_{c}}{L}}$$  \hspace{1cm} (12)

$$2\xi\omega_{n} = \frac{2K_{p2}V_{c}}{L}$$  \hspace{1cm} (13)

Assuming that the natural frequency for system $\omega_{n}$ is equal to $1/m$ of switching frequency, and the critical damping mode of $\xi=1$ is considered, system parameters are designed as follows:

$$K_{I2} = \frac{(2\pi)^2f_{s}^2L}{2mV_{c}}$$  \hspace{1cm} (14)

$$K_{p2} = \frac{2\pi f_{s}L}{mV_{c}}$$  \hspace{1cm} (15)

To design voltage control (outer loop), according to equation (6), the following equation can be obtained in the Laplace domain:

$$G_{V}(S) = \frac{V_{r}(S)}{I_{f}(S)} = -\frac{1}{CS}$$  \hspace{1cm} (16)
The aim of designing voltage controller is to fix DC link voltage in its reference value and generate the reference current necessary for the inner loop. The active filter current can be obtained in terms of the load current and the network current, as follows:

\[ i_F(t) = i_S(t) - i_L(t) \]

Network current can be obtained by a PI controller in the outer loop (voltage loop). The conversion function of this controller is considered as the following:

\[ G_{PI}(S) = \frac{K_{pI}S + K_{iI}}{S} \]

Because the reactive power taken from the network should be zero, the network current should be in the same phase with the network voltage. The diagram block for voltage control loop is shown in Figure 5.

**Figure (5) the outer loop (voltage loop) controller**

In the diagram block (5), to design PI controller, we should convert multiply block to addition block. Assuming that we are in steady state and \( W_0 \) is the steady-state value of \( W(t) \), the multiplier block can be simplified as the following:

\[ W(t)v_S(t) \approx W_{SS}v_S(t) \]

On the other hand, the \( i_S(t) \) current should not include DC component. As a result, the value of \( W_0 \) in the addition block should be added with a negative sign, to eliminate its effect from \( W(t) \) when converting multiply block to addition block and does not produce DC component in \( i_S(t) \). Thus, the control diagram block to design PI is shown in figure (6). As a result, the conversion function becomes:

\[
V_C(S) = \frac{G_v(S)}{1 - G_{PI}(S)G_V(S)} \left( W_oV_S(S) - \frac{W_o}{S} + I_L(S) \right) - \frac{G_v(S)G_V(S)}{1 - G_{PI}(S)G_V(S)} V_{C_{ref}}(S) \\
= -\frac{S}{CS^2 + K_{pI}S + K_{iI}} \left( W_oV_S(S) - \frac{W_o}{S} + I_L(S) \right) + \frac{K_{pI}S + K_{iI}}{CS^2 + K_{pI}S + K_{iI}} V_{C_{ref}}(S)
\]

The characteristic equation of the conversion function for equation (20) is as follows:

\[ \Delta(s) = CS^2 + K_{pI}S + K_{iI} \]

**Figure (6) A simplified diagram block to design PI controller in the outer loop controller**
By comparison with equation (21) with a classic form characterized by conversion function, that is
\[ \Delta(s) = S^2 + 2\zeta\omega_n S + \omega_n^2, \]
the following equations are obtained for the natural frequency and damping rate of the system:
\[ \omega_n = \sqrt{\frac{K_{I 1}}{C}} \]
22
\[ K_{P 1} = \frac{4\pi f_c C}{n} \]
23
Assuming that the natural frequency of the system \( \omega_n \) is equal to \( 1/n \) of voltage source frequency (in the outer loop, a low-frequency DC can be tracked; as a result, its bandwidth can be considered a quantitative number), the critical damping mode of \( \zeta = 1 \) is considered, system parameters are designed as follows:
\[ K_{I 1} = \frac{(2\pi)^2 f_s^2 C}{n^2} \]
24
\[ K_{P 1} = \frac{4\pi f_c C}{n} \]
25
The control diagram block of the total system is shown in figure (7).

![Figure (7) The general control diagram block of the single-phase active filter](image)

5. Simulation
The effective value and frequency of network voltage are respectively 50Hz and 50Hz, and values \( L = 500\mu H \) and \( C = 470\mu F \) are chosen. DC link voltage is considered to be 200V. The switching frequency is \( f_s = 40\text{KHz} \). The natural frequency of the loop current is equal to \( 1/5 \) times of switching frequency and as a result, \( m = 5 \). Using equations (14) and (15), we have:
\[ K_{I 2} = \frac{(2\pi)^2 f_s^2 L}{2m V_C} = \frac{(2\pi)^2 \times 40000^2 \times 500 \times 10^{-6}}{2 \times 5^2 \times 200} = 3158.3 \]
26
\[ K_{P 2} = \frac{2\pi f_s L}{m V_C} = \frac{2\pi \times 40000 \times 500 \times 10^{-6}}{5 \times 200} = 0.1257 \]
27
As a result, the loop current controller will be achieved as follows:
\[ G_{P 2}(S) = 0.1257 + \frac{3158.3}{S} \]
The bandwidth for voltage control loop is considered to be $1/10$ times of network frequency and thus $n=10$. Using equations (24) and (25), we will have:

$$K_{I1} = \frac{(2\pi)^2 f_s^2 C}{n^2} = \frac{(2\pi)^2 \times 50^2 \times 470 \times 10^{-6}}{10^2} = 3.4568$$

$$K_{P1} = \frac{4\pi f_s C}{n} = \frac{4\pi \times 50 \times 470 \times 10^{-6}}{10} = 0.22$$

Finally, the voltage loop controller will be calculated as follows:

$$G_{P2}(S) = 0.0295 + \frac{0.4638}{S}$$

As a result, the simulated system parameters are as follows:

- $f_s = 40\, KHz, \quad V_s = 110\, \text{V} - \text{RMS}$
- $L = 500\, \mu\text{H}, \quad C = 3500\, \mu\text{F}$
- $U_{DC} = 200\, \text{V}$
- $K_{I2} = 3158.3, \quad K_{P2} = 0.1257$
- $K_{I1} = 3.4568, \quad K_{P1} = 0.2$
- $R_L = 5\, \Omega, \quad C_f = 1000\, \mu\text{F}, \quad L_f = 5\, \text{mH}$

The following figures show the simulation results for the system. As shown in figure (9), load current is strongly harmonic and the amount of its harmonic distortion is $31/3\%$ (figure 12), but after being compensated by active filter, network currents is substantially approached to the sinusoidal form that is evident in figure (11), and the amount of its harmonic distortion is $5\%$ (Figure 13). as can be seen, the active filter injects the harmonics and reactive power required for the load, and the network only injects the sinusoidal and active component (Figure 8).

![Figure (8) network voltage](image)

![Figure (9) current load](image)
The harmonic spectrum of load current and network current:

- Fundamental (50Hz) = 42.33, THD= 31.31%
- Fundamental (50Hz) = 37.29, THD= 5.00%

DC link voltage is shown in figure (14). DC link voltage is stabilized in its reference value of 200 volt after some period.
Simulation results for the system can be shown for dynamic and abrupt changes in the load. As shown in figures (15) and (16), after load changes, the controller works and active filter can compensate the harmonic current and reactive power of load. According to figure 17, DC link voltage reaches its value again after a while and will remain at that value.

Figure (15) the load current simulation for dynamic changes

Figure (16) the network current simulation for dynamic changes

Figure (17) the DC link voltage changes for load dynamic change

Changes in active and reactive power of load and network for dynamic load changes are given:
CONCLUSION
In this paper, a single-phase active filter is modeled and mathematically controlled to improve the power quality. At the times of switching the key, equivalent circuits were obtained and the system was modeled by writing space state equations. With a mathematical model, the classic double-loop control system was designed for active filter. The inner loop with relatively high bandwidth is responsible to control active filter current, and the outer loop is responsible to control DC link voltage and generate a reference current for the inner loop. Using the conversion functions obtained from the mathematical models, controllers were designed for both inner and outer loops. To test the performance of the proposed system, a designed sample was simulated in MATLAB SIMULINK software. Using the designed controller, our desired active filter was enabled to compensate harmonic currents and reactive power of load, even during the sudden changes in the loads, and the DC link voltage was also stabilized in its reference value under any circumstances.

REFERENCES