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ORIGINAL ARTICLE

Application of Lyapunov Exponent on the Predictability of Temperature over an Equatorial Station-Chennai

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ABSTRACT
The global mean temperature near the surface of the Earth is often used as the primary indicator of climate change. The recent hiatus in global surface temperature rise is of considerable interest. In this work, we tried to analyze the hiatus for an equator station, Chennai, using the average daily temperature records for 25 years (1989 – 2013). The purpose of this contribution is to try to detect deterministic chaos in the temperature series in order to investigate their chaotic behavior, and determine their error doubling time using the Lyapunov exponent method. The data set is split into five sectors from S1 to S5 each with five years duration. Estimates of predictability of the average daily temperature, in this study, brings out clear evidence of significant decrease in predictability over the equatorial Chennai region of Tamil Nadu state during the past couple of sectors compared to earlier parts of this century. The error doubling time over the equatorial Chennai region of Tamil Nadu was close to 12.9 hours during the first sector of the century, it has decreased to 3.5 hours for the next sector S2. This doubling has sustained for the sectors 3 and 4 to 2.4 hours. The sector 5 gives a unique result with an increase in error doubling time (5.56 hours). This result is attributed to the global warming hiatus, which may have made the temperature system less complex and increased the prediction period of temperature. The corresponding largest Lyapunov exponent for all the sectors shows positive values which determine the chaotic behavior of average daily temperature of Chennai region.
Keywords: hiatus, equatorial station, predictability, chaos, Lyapunov exponent, error doubling time

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Introduction

In recent years, the increase in near-surface global mean temperatures has emerged considerably smaller than many had expected. We investigate whether this can be studied by climate change scenarios. Estimates of the observed global warming in the recent 15-year period 1998-2012 are significantly lower than the average warming of 0.02^0C/year observed in the past thirty years 1970-2000 [1]. Researchers have followed various leads in recent years, concentrating mainly on a trio of factors: the Sun [2] atmospheric aerosol particles [3] and the oceans. These factors could be primary drivers of the hiatus, but reports published in the past few years reported that their effects were likely to be relatively small [4]. In this paper, we have tried to calculate the error doubling time for the prediction of temperature for the region of Chennai. Such an estimate will require a measurement of the growth rate of initial uncertainties. Lyapunov exponents are the measure of sensitivity to initial conditions with the magnitude of the exponent reflecting the time scale at which the system becomes unpredictable. In any liberal dynamical system, there exists at least one negative exponent and the sum of all of the exponents is negative. In the case of chaotic systems, there will be at least one positive Lyapunov exponent. Various methods have been proposed to estimate the largest Lyapunov exponent from the observed single time series. The predictability of chaotic time series could be achieved by the method of Lyapunov exponent. To date, a lot of attention has been devoted to analyzing various natural processes and elements by means of deterministic chaos approach [5-12].

The study and the analysis of nonlinear systems is a crucial issue in the wide variety of fields, and no model of a real system is truly linear, but some aspects are studied as linear approximations to the real models. Various methods have been developed to abstract different kinds of information in the data besides time series plots, such as autocorrelation method, power spectrums, Hurst and principal component analysis (PCA) and so on. The autocorrelation analysis [13] is a suitable mathematical tool for displaying the temporal dependency and finding repeating patterns in the time series. The Hurst coefficient weighs the phenomenon of persistence and reveals the long-term memory of the data. PCA [14] can be thought of revealing the internal structure of the data in a way which best explains the variance of the data. However, it is a linear dimensionality reduction method that cannot adequately represent nonlinear relations. Therefore, contrary nonlinear time series analysis methods based on the chaos and dynamic theory were refined, such as the correlation dimension (CD) method [15], Kolmogorov entropy, recurrence quantification analysis and the Lyapunov exponent [16]. These methods motivate scientists to explore the hidden information in the data from the special view of the "complexity" underlying the system dynamics.

Study area

Chennai is one of the major metropolitan cities, situated on the south east coast of India. The city is located in the state of Tamil Nadu. Chennai is 25.6 km in length and extends inland to about 11 Km and the total area is 174 Km^2. The geographical coordinates of the study area are 13°10′04′′N latitude and 80°15′43′′ E longitude and it is located at an average altitude of 6.7 meters from the sea level as in Figure 1. Most of the water resources have been polluted in recent years, ultimately due to rapid urbanized, industrialized, commercialized factors which includes migration. Chennai has a tropical, wet and dry climate. Since it is situated along the coast, there is very less seasonal variation in the temperature.

DATA AND METHODOLOGY

The importance of this work is to analyze the hiatus for an equatorial station like Chennai, in India using the average daily temperature records for 25 years. The daily average temperature data on the equatorial station, Chennai was obtained from the website www.tutiempo.net/en/Climate/Madras_Minambakkam for a period of twenty-five years from 1989 to 2013. With our objective of finding change in the predictability of temperature over Chennai, the total data were divided into five sectors of 5 years each, namely 1989-1993 (S1), 1994– 1998 (S2), 1999-2003 (S3), 2004-2008 (S4), 2009-2013 (S5). The embedding dimension (m) and time delay (τ) was calculated using the Visual Recurrence Analysis software version 4.9. After obtaining the above values, the largest positive Lyapunov exponent was calculated with the help of Matlab algorithm.

Lyapunov Exponent

Methods to estimate Lyapunov exponents include Wolf method, Jocobian method and the small amount of data methods. The small amount of data, method is more reliable for small data sets, has small computation and is relatively easy to operate. The small data set method is proposed by Rosenstein M. T, Collins J.J. and Deluca C. J. as in [17]. This method is used to calculate largest Lyapunov exponent based on the real time series. This method needs the calculation of the phase space reconstruction.

Phase space reconstruction

From the single time series of temperature data, the phase space of evolution of the system is reconstructed by the method of time delay [18]. Phase space reconstruction is necessary condition for calculating Lyapunov exponents of time series. Phase space reconstruction theorem considers that the reconstructed phase space with embedding dimension m ($m \geq 2D + 1$,D is correlation dimension of the system) and delay time can present character of the whole system. We need to reconstruct the phase space of observation sequence and recover the form of chaotic attractors in high-dimensional phase space in order to analyze and predict chaotic dynamical system. Phase space reconstruction needs an estimation of correlation dimension and time delay of chaotic time series. Therefore, to study the chaotic dynamical system, we must first determine the correlation dimension and time delay. The methods to select time delay include serial correlation such as average mutual information (AMI) method and the embedding dimension is estimated using the false nearest neighbor (FNN) method in the preparation for estimating the largest positive Lyapunov exponent over Chennai is shown in **Figure 2 and 3** respectively.

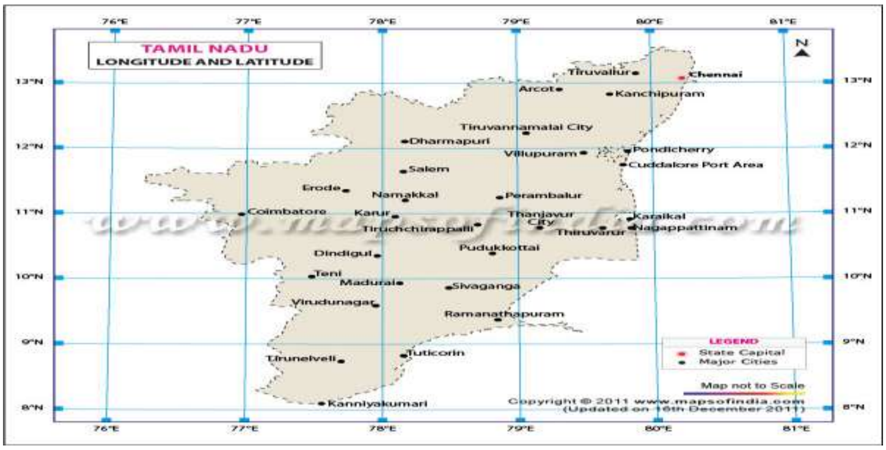
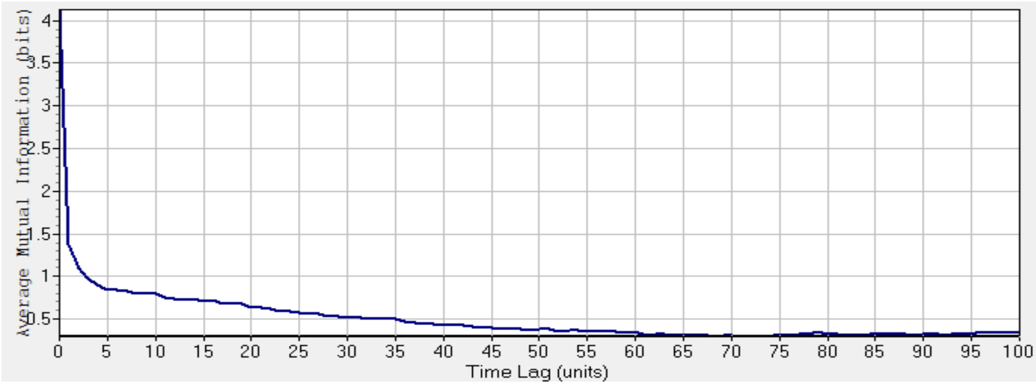
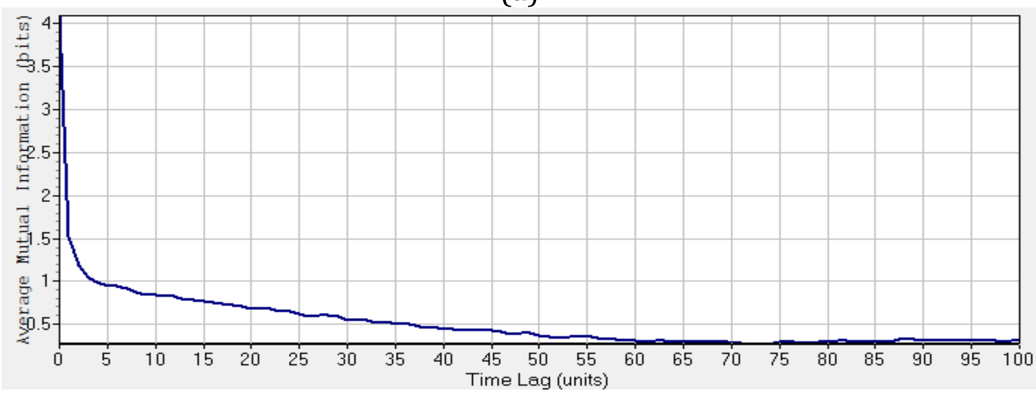


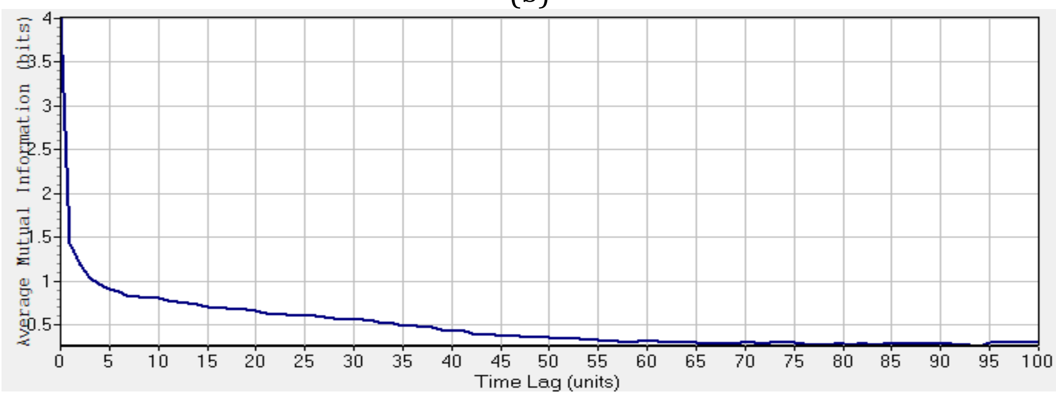
Figure 1. Location of the study area - Chennai in Tamil Nadu, India



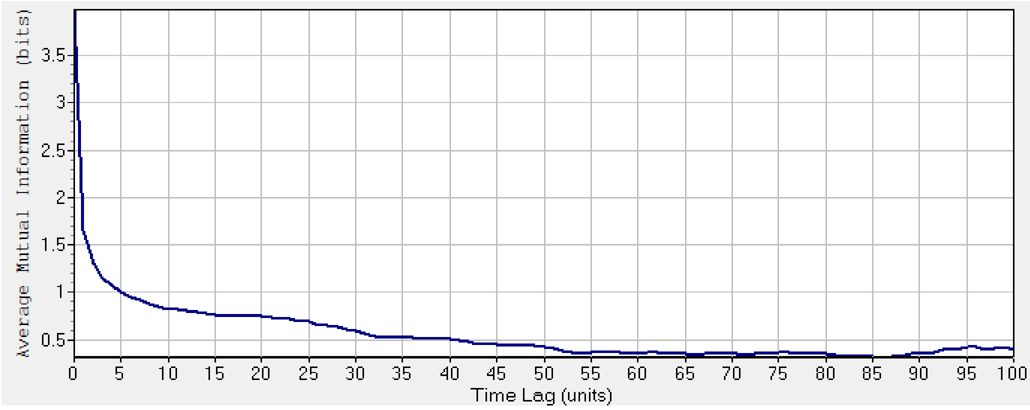
(a)



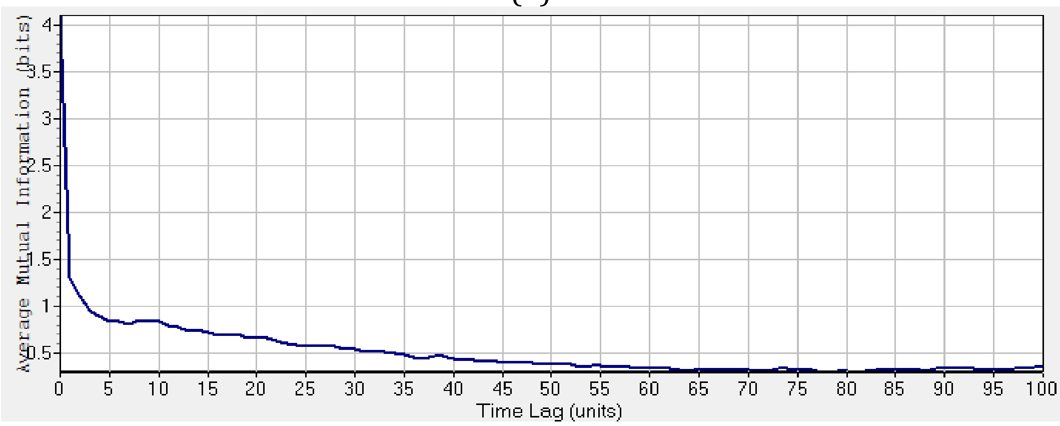
(b)



(c)

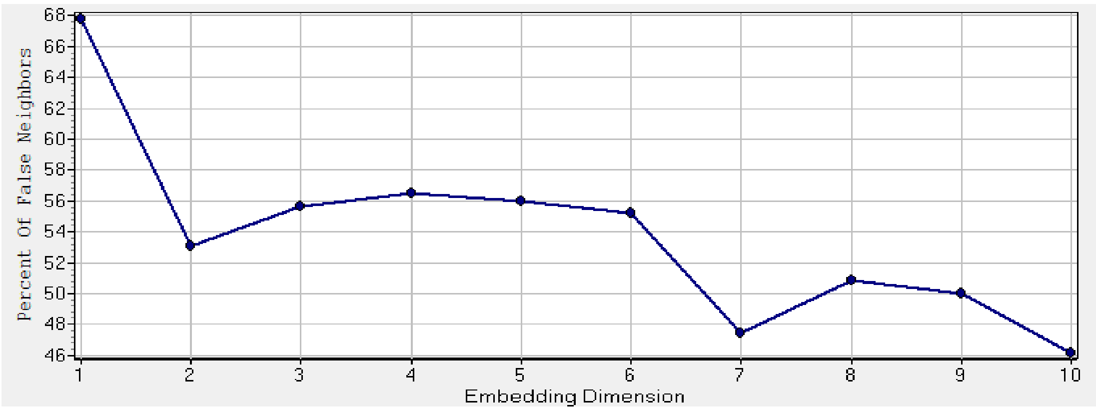


(d)

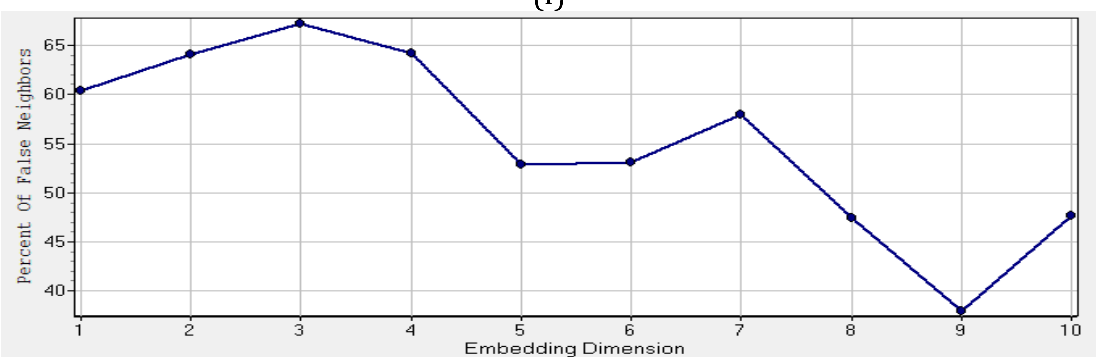


(e)

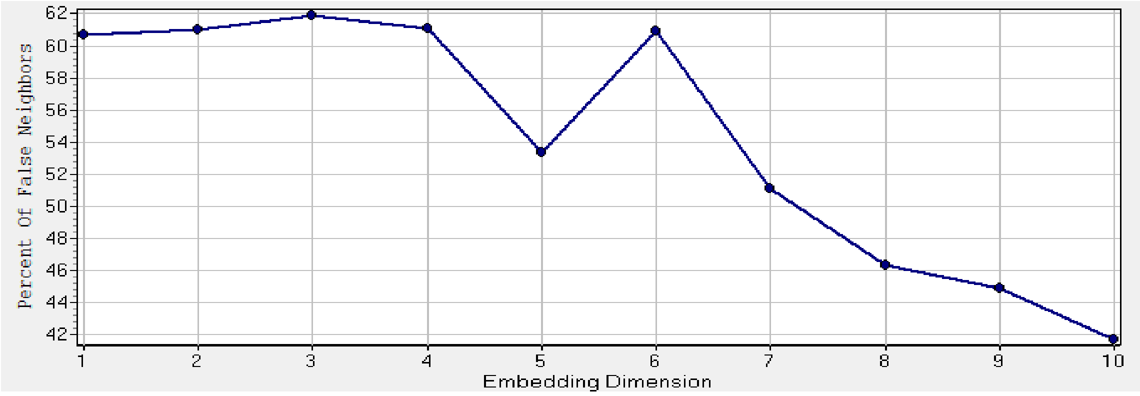
Figure 2 . (a), (b), (c), (d), (e) Average mutual information method for calculating time delay for all the five sectors S1-S5



(f)

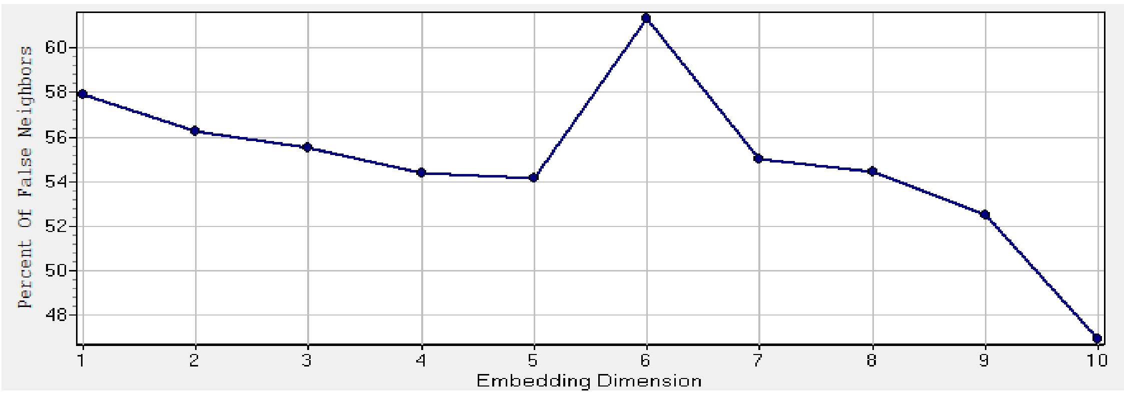


(g)



(h)

(i)



(j)

Figure.3 (f), (g), (h), (i), (j) False Nearest Neighbor method (FNN) for calculating the embedding dimension method for all the five sectors S1-S5.

RESULTS AND DISCUSSION

Using the average daily temperature data for the Chennai region, the period of error doubling was calculated using the largest positive Lyapunov exponent (λ) for all the five sectors and their corresponding embedding dimension, the time delay is also tabulated in **Table 1**. The results clearly conform the evidence of chaotic nature in the average daily temperature over Chennai, India as the largest Lyapunov exponent is positive for the five sectors. There is a significant decrease in error doubling time for the four sectors (S1-S4) and increase in error doubling time for the last sector (S5). While the error doubling time over Chennai sustained for 2.4 hours during the third and fourth sector and it has increased to 5.5 hours during the last sector, throwing up new challenges in predicting the daily average temperature. The hiatus of average daily temperature was observed during S3 and S4 sectors for 10 years with an increase in error doubling time in S5 sector. This can be due to the internal natural climate variability or due to the external forcing processes in addition to anthropogenic forcing or may be due to the climate model sensitivities to external anthropogenic forcing is too high.

Sector	S1	S2	S3	S4	S5
Year	1989-1993	1994- 1998	1999-2003	2004- 2008	2009 – 2013
Embedding dimension(m)	10	9	10	10	10
Time delay (τ)	13	9	5	9	16
Largest Lyapunov exponent (λ)	1.848	6.8257	9.7389	9.8445	4.31284
Period	0.5411	0.1465	0.1027	0.1016	0.2319
Predictability time or error doubling time	12.99	3.52	2.46	2.44	5.56

Table 1 Results of embedding dimension, time delay, largest lyapunov exponent, period and the predictability time for the daily average temperature for Chennai for all the five sectors from 1989-2013.

CONCLUSION

Estimates of predictability of daily average temperature of an equatorial station, Chennai, brings out clear evidence of chaos as the largest Lyapunov exponent is positive in all the five sectors. The hiatus of average daily temperature was observed significantly during S3 and S4 sectors for 10 years with an increase in error doubling time was identified in S5 sector. The sustained global warming trend which has not risen at least greater than its long term average could have reduced the complex nature of the system under study enabling an increased predicting time. The error doubling period of the daily average temperature has increased from 2.4 (S3, S4) to 5.5 (S5). This result is attributed to the global warming hiatus, which has made the temperature system less complex and increased the prediction period of temperature. We continue our research further as the current observations are not detailed enough or of long enough duration to provide definitive answers on the causes of the recent hiatus, and therefore do not enable us to conclude. These are major scientific challenges that the research community is actively continuing, resulting on exploration and experimentation using various methods of theory, models and observations.

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