Reducing Air Traffic Delay Cost using Dynamic Network Flow with Random Capacity

Sobhan Mostafayi1, Morteza Dahmardeh2, Kurosh Mokhtari3

1,3Department Of Industrial Engineering, Birjand University Of Technology, Birjand, Iran
2Department of Industrial Engineering, Ferdosi University Of Mashhad, International Branch, Iran

Sobhan.mostafayi@gmail.com, Mtz.dahmardeh@gmail.com, kuroshmokhtari@yahoo.com

ABSTRACT
The final goal of air traffic control is to establish efficient transference of airplanes while maintaining airborne operation safety. The main objective, however, is safety which may under no circumstances be neglected. Air traffic delays occur when demands for landing or takeoff exceed the existing capacity. The effects of these delays may be reduced via increase in capacity or decrease in demand. Capacity increase is of course a good but long-term solution that is facilitated by building new airports, expanding existing airports, and new landing methods. Therefore, a short-term decision for reducing delays in view of current facilities requires a tactical optimization model. In this paper, the framework of this model, which is a generalization of the dynamic network flow model with random capacity, is presented.

Keywords: Air traffic control, dynamic network flow, random programming, delay costs.

INTRODUCTION
Traffic can gradually turn into a serious problem in most airports. The annual delay cost caused by air traffic in Europe is estimated to be 5 billion dollars in 1989. The same cost was approximately 2 billion dollars for American airlines. Since the final profit of the American airline industry rarely exceeds one billion dollar, the importance of the traffic problem may be understood. The final goal of air traffic control (ATC) is to establish efficient transference of airplanes while maintaining airborne operation safety through international airborne systems (NAC). The main objective, however, is safety which may under no circumstances be neglected. The main reason for traffic is limitation in capacity. In fact, air traffic delays occur when demands for landing or takeoff exceed the existing capacity. Generally, NAC capacity is sufficient under proper weather conditions. Nevertheless, undesirable weather has significant effects on NAC capacity. According to the Federal Aviation Administration (FAA) statistics in 1993, 72 percent of delays lasting over 15 minutes are caused by undesirable weather conditions. Generally, NAC capacity is sufficient under proper weather conditions. Nevertheless, undesirable weather has significant effects on NAC capacity. According to the Federal Aviation Administration (FAA) statistics in 1993, 72 percent of delays lasting over 15 minutes are caused by undesirable weather conditions. Moderate weather causes traffic in the entrance of destination airports. Air Transport Association (ATA) estimated ATC delays for ATA airlines to be 2.3 billion dollars, together with 1.35-billion-dollar cost caused by passenger delays. Hence, a small improvement in ATC delays leads to a considerable decrease in delays cost. The following question is posed: How may delays be reduced? A long-term measure for reducing delays is increase in landing capacity which is facilitated by building new airports, adding runways to the existing airports, using larger airplanes and advanced landing methods. Building new airports is, however, very difficult and expensive in a way that only six out of twenty airports with most delays in 1993 developed plans for building new runways by 2000. Moreover, advanced landing methods and employing electronic automatic machines require training and steering equipment both of which are costly for FAA and the airline industry. On the other hand, there were around 4,497,369 flights in the American airlines in 1982. With a 38.3 increase, this number reached 6,668,523. FAA predicts that the number of flights will experience another 38% increase until 2005. Thus, although efforts for building new facilities continue, these facilities are merely effective for adaptation with traffic growth. New techniques are, therefore, required so that delays may be reduced in the short term and given the existing facilities. It is certain that...
delays cannot be completely eliminated; however, the effects of delays may be reduced by employing managerial techniques of the traffic flow.

There are three managerial techniques for reducing delays which will be introduced and presented in the subsequent sections. The third technique is more efficient. Furthermore, the first and third techniques are executable by employing the third technique. These three techniques are as follows:

A. No Flow Control Policy (NFC)

As a matter of fact, this technique lacks any management of traffic flow. All airplanes fly according to schedule with no management and when they approach the destination airport, the runway capacity may not be to the extent that all airplanes can land. Thus, most airplanes have to wait in the long airway line and circle around the traffic station called the “waiting zone.” Nonetheless, maintaining airplanes in this zone is illogical because of extra fuel consumption costs, air pollution, and increase in dangers and is contradictory to ATC’s main objective being increase in safety.

B. First-Come First-Served Innovative Policy (FCFS)

Air traffic control system command center (ATCSCC) recently makes practical managerial decisions for NAC. ATCSCC considers landing times in accordance with the first-come first-served innovative policy and allocates ground delays to flights for conformity with these landing times. The main reason why this policy is adopted is the fact that ground delays have less cost compared with air delays. It also increases safety. The present capacity is specified in this technique at any place and there is a certain prediction for the capacity of the next airport. If the airport capacity is in accordance with the predicted capacity, this will be an efficient policy. However, since only one prediction is made, the prediction may not be very accurate. In this event, there is a negative effect on response quality. Of course, this issue that determines how long each airplane remains in the air or with what velocity it moves in a way that as soon as it arrives at the destination, it would not have to stay in the air for long, thus minimizing ground and air delay costs is called the ground-holding problem (GHP).

C. Multiple-Scenario Dynamic Network Flow Technique

The framework of this technique is based on a place-time network flow model, some of the arcs of which have uncertain capacity. Uncertainty is defined via a multiple-scenario approach, where each scenario demonstrates a possible state of the capacity vector. The scenario method has two important features. Firstly, it is reliable, because it allows the decision maker to select scenarios. For example, the decision maker can generate a scenario that involves “the best guess,” “the best case,” or “the worst case” possible from airport capacities. Secondly, since the probability distribution of the next airports’ capacity is practically unknown, the scenario approach can be useful. Besides, predicting the capacities of next airports’ capacities at specified times requires precise prediction of weather conditions, whereas certain prediction of weather is impossible via present technology. This makes the scenario approach more appealing. This paper is organized as follows. The overall structure of random dynamic network flow is introduced in the second section. In the third section, the air traffic management problem is modeled in the framework of this network. In the fourth section, the required data for examining the multiple-scenario dynamic network flow technique is introduced. In the fifth section, the responses obtained from different managerial models are analyzed using statistical simulation. Finally, in the sixth section, experimentation results of the MSDNF model are investigated in comparison with other models.

2. Random Dynamic Network Flow

The random dynamic network flow problem deals with cases where some or all network parameters are expressed by random (or probabilistic) variables instead of definite quantities. The main idea behind solving this type of random problems is to convert them into equivalent definite problems. To do so, random programming method with random constraints or scenario aggregation modeling may be used. Random programming with random constraints is employed in solving problems comprising of random constraints, i.e. constraints for which there is a specific probability for rejection. However, this method cannot be used when probability distribution is unknown. To resolve this problem, scenario aggregation modeling is expressed for the dynamic network flow problem with random arc capacity. Randomness is modeled by employing a sample instead of distribution. This model can be particularly useful when random variables among different cycles are time-dependent.

**Definition 1.** The network flow $G=(V,A)$ is dynamic when each node $i \in V$ has time $t(i)$ and for each arc $(i,j) \in A$ we have $t(i) < t(j)$.

According to the dynamic network flow definition, each node $i$ has time $t(i)$. Supposing that $\eta = \{1,2,\ldots,T\}$ is the set of nodes’ periods of time and $U_t$ is the capacity vector of arcs in the $t$-th period, $S = \{U_{t1}^1, U_{t2}^2, \ldots, U_{tT}^T\}$ is called a scenario that indicates one possible state of the network capacity.
vector. In the scenario aggregation model, it is supposed that there is a finite number of scenarios each of which happen with a specific probability.

Consider network flow \( G = (V, A) \) where \( V = \{1, 2, \ldots, n\} \) is the set of network nodes, \( \eta = \{1, 2, \ldots, T\} \) is the set of nodes’ periods of time and \( A \) is the set of network arcs. In this network, arcs’ capacity has a discrete random variable.

Suppose that \( \Omega = \{1, \ldots, K\} \) is the set of scenario indices, \( u^k \) is the capacity vector of scenario \( k \) and \( p_k \) is the probability of occurrence of scenario \( k \). When arc \( a \) has no finite capacity limit under scenario \( k \), then \( u^k_a = \infty \). It is supposed that the capacity of arc \( a = (i, j) \) is specified at the start time of arc \( a \), i.e. \( t(a) \).

As a matter of fact, randomness is described by a multiple-scenario approach. These scenarios are time-dependent and each scenario specifies one possible state of the arc capacity vector. Thus, time-dependency of scenarios is explained before presenting the model.

Suppose \( A_t = \{a \in A : t(a) \leq t\} \) is a subset of \( A \) starting before time \( t \). For each \( k \neq k' \), \( k \) and \( k' \) scenarios are said to be indistinguishable, when for each \( a \in A_t \), the capacity of arc \( a \) is the same under both \( k \) and \( k' \) scenarios. If \( \tau(k, k') \) is the latest time when \( k \) and \( k' \) scenarios become indistinguishable, then \( \tau(k, k') = \max \{ t : u^k_a = u^{k'}_a \, \forall a \in A_t \} \). This means that the capacity of each arc the start time of which is less than or equal to \( \tau(k, k') \) is identical under \( k \) and \( k' \) scenarios and there is at least one arc with the start time \( 1+ \tau(k, k') \) that has a different capacity under \( k \) and \( k' \) scenarios.

Suppose \( x^k \) is the flow vector under scenario \( k \), \( c \) is the cost vector and \( N \) is the node-arc incidence matrix of network \( G \). Node 1 and node \( n \) are selected to be origin and unique destination, respectively. In addition, it is supposed that there is exactly \( v \) units of offer in node 1 and \( v \) units of demand in node \( n \) in the problem. Hence, \( b_n = v, b_1 = -v \) and for each \( i \in V \setminus \{1, n\}, b_i = 0 \).

With this description, the scenario aggregation model corresponding to the dynamic network flow is as follows:

\[
\begin{align*}
\text{min} & \quad \sum_k p_k (c x^k) \\
\text{s.t.} & \quad N x^k = b \quad \forall k \quad (1-1) \\
& \quad x^k \leq u^k \quad \forall k \quad (1) \\
& \quad x_a^k - x_a^{k'} = 0 \quad \forall a \in A; \forall k, k': t(a) \leq \tau(k, k') \quad (3-1) \\
& \quad x^k \geq 0 \quad \forall k \quad (4-1)
\end{align*}
\]

In (1), (1-1) is flow conservation constraints, (2-1) is capacity constraints, (3-1) is unpredictable constraints, and (4-1) is non-negative constraints. Unpredictable constraints indicate that when two or more scenarios are indistinguishable, a single decision cannot predict which scenario happens. Without loss of generality, it is supposed that all scenarios are indistinguishable in the first period of time. For example, the structure of a network flow problem with two scenarios may be found in Paradigm 1.

\[
\begin{align*}
\text{min} & \quad p_1 (c_x x_1 + c_y y_1) + p_2 (c_x x_2 + c_y y_2) \\
\text{s.t.} & \quad N_x x_1 + N_y y_1 = b \quad (2) \\
& \quad N_x x_2 + N_y y_2 = b \quad (3) \\
& \quad 0 \leq x_1 \leq k_x \quad (4) \\
& \quad 0 \leq y_1 \leq k_{y1} \quad (5)
\end{align*}
\]
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\[
x_2 \leq k_x \quad (6)
\]
\[
y_2 \leq k_{y_2} \quad (7)
\]
\[
x_1 - x_2 = 0 \quad (8)
\]

Paradigm 1: Two-scenario dynamic network flow problem

Capacity scenarios are \((k_x, k_{y_1})\) and \((k_x, k_{y_2})\). (2), (4), and (5) constraints as well as \(x_1\) and \(y_1\) variables correspond to the network flow subproblem pertaining to the first scenario. (3), (6), and (7) constraints as well as \(x_2\) and \(y_2\) variables correspond to the network flow subproblem pertaining to the second scenario.

As can be seen, scenarios are identical at the beginning. Therefore, the decision made under these two scenarios should reflect the unpredictability of scenarios in the first period of time. In fact, unpredictable constraints in (8) guarantee that the decisions of the first period of time are identical.

Glockner expressed the general structure of the multiple-scenario network flow problem in 2000 [3]. He then put forward algorithms for solving this problem in 2001 [2].


In this section, the approach is based on the optimization of air traffic delays on a place-time dynamic network flow model. Different types of managerial decisions such as airplane velocity increase or decrease en route can be simultaneously incorporated in the dynamic network flow model. This model should be solved by a proper programming problem, because delayed decisions may not be made for a fraction of the airplane.

Suppose \(L = \{1, 2, \ldots, l\}\) is a set of different locations including the departure airport, the main runway of the destination airport, the waiting zone, and a part of the air space and \(\eta = \{1, 2, \ldots, T\}\) is an ordered set of time periods. For instance, \(\eta\) can be a set of sixty four 15-minute time periods showing a part of day from 7 A.M. to 11 P.M. (16 hours) during which most of flights take place. There is density in some locations in \(L\) indicating the main runway of the destination airport and the waiting zone. For each dense location \(l_i\), the set \(L\) is divided into \(l_i^1\) and \(l_i^2\) locations.

A set of \(V = L \times \eta\) nodes is created, where each node \(n_j = (l_j, t_j)\) points out to location \(l_j\) at time \(t_j\) and each arc \(a \in A\) is in form \(a = (l_{i_1}, t_{j_1}), (l_{i_2}, t_{j_2})\) where for each \(a \in A, t_{j_2} > t_{j_1}\).

There are five types of arcs in this network including:

a. **Flight arc**: Whenever flight from location \(l_{i_1}\) to location \(l_{i_2}\) requires time \(t_{j_2} - t_{j_1}\), flight arc is defined from node \((l_{i_1}, t_{j_1})\) to node \((l_{i_2}, t_{j_2})\).

b. **Delay arc**: Whenever there is a delay for a flight in location \(l_j\), this arc is defined from node \((l_j, t_j)\) to node \((l_j, t_j + 1)\).

c. **Limitation arc**: For dense location \(l_i\), limitation arc is defined as an arc from node \((l_{i_1}, t_{j_1})\) to node \((l_{i_2}, t_{j_2})\). In fact, limitation arcs show the degree of traffic in the waiting zone and have indefinite capacity.

d. **Destination arc**: Whenever \(l_j\) is a destination, this arc is defined from node \((l_j, t_j)\) to node \((l_j, T + 1)\).

e. **Extra arc**: This arc is defined from node \((l_{i_1}, T)\) to node \((l_{i_2}, T + 1)\).

Extra arc is used when an airplane’s delay is longer than a traffic period. Destination and extra arcs are of artificial type. Limitation arcs have finite capacity so that the degree of traffic in location \(l_i\) could be modeled. The five types of arcs in the network may be seen in Table 1.

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**Table 1**

<table>
<thead>
<tr>
<th>Arc Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flight</td>
<td>From location (l_{i_1}) to location (l_{i_2}) requiring (t_{j_2} - t_{j_1})</td>
</tr>
<tr>
<td>Delay</td>
<td>From node ((l_j, t_j)) to node ((l_j, t_j + 1))</td>
</tr>
<tr>
<td>Limitation</td>
<td>From node ((l_{i_1}, t_{j_1})) to node ((l_{i_2}, t_{j_2}))</td>
</tr>
<tr>
<td>Destination</td>
<td>From node ((l_j, t_j)) to node ((l_j, T + 1))</td>
</tr>
<tr>
<td>Extra</td>
<td>From node ((l_{i_1}, T)) to node ((l_{i_2}, T + 1))</td>
</tr>
</tbody>
</table>

---

**Figure 1**

Dynamic Network Flow Model

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**Figure 2**

Scenario Network Flow Problem
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Flights correspond to flow in network. Each flight corresponds to an object (a pair of origin-destination). The origin node of network corresponds to the start location and departure time of flight. Moreover, each flight (object) has a specific destination, but may have several different candidates in landing times. Each destination $l_e$ is connected to the final node $(l_e,T+1)$ via arc $(l_e,t),(l_e,T+1)$. These destination arcs have infinite capacity and zero cost and guarantee that each flight arrives at its destination. Additionally, there are extra arcs $(l_i,T),(l_i,T+1)$ with infinite capacity for each pair $l_i$ and $l_e$. The combination of delay and extra arcs ensures that each problem has an answer despite the randomness of capacity. An example for the problem of ground delays may be found in Figure 1. In this example, D1, D2, and D3 show cities that are 1 hour, 2 and 3 hours away from the destination, respectively. Flows from D1, D2, and D3 nodes show flights. Airplanes fly to the dense airport A. Nodes of A are split to $A'$ and $A''$ nodes, where they indicate the waiting zone and the main runway of the airport, respectively. Thus, arcs from $A'$ to $A''$ show the process of landing. There are two types of delay arcs in the network showing air and ground delays. It should be noted that (D1,8), (D2,7), and (D3,6) nodes are not origin nodes. These nodes together with extra arcs show a situation when flights take place at a time later than the traffic period. The final destination is a square node which is in fact node $(l_1,10)$. For instance, consider a two-hour flight at 3 P.M. This flight is demonstrated as a flow unit from node (D2,3) to $(l_1,10)$. It is random limitation.}

### Table 1: Types of arcs

<table>
<thead>
<tr>
<th>Arc type</th>
<th>Form</th>
<th>Cost</th>
<th>Capacity</th>
<th>Considerations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flight</td>
<td>$(l_i,t^1),(l_i,t^2)$</td>
<td>0</td>
<td>Infinite</td>
<td>$l_i \neq l_e$, $t_j^2 &gt; t_j^1$</td>
</tr>
<tr>
<td>Delay</td>
<td>$(l_i,t^1),(l_i,t^2)$</td>
<td>Usually positive</td>
<td>Infinite</td>
<td></td>
</tr>
<tr>
<td>Limitation</td>
<td>$(l_i,t^1),(l_i,t^2)$</td>
<td>0</td>
<td>Finite</td>
<td>It is random</td>
</tr>
<tr>
<td>Destination</td>
<td>$(l_i,t^1),(l_i,T+1)$</td>
<td>0</td>
<td>Infinite</td>
<td>$l_i$ is a destination</td>
</tr>
<tr>
<td>Extra</td>
<td>$(l_i,T),(l_i,T+1)$</td>
<td>0</td>
<td>Infinite</td>
<td>$l_i = l_e$</td>
</tr>
</tbody>
</table>

If runway capacity is insufficient, the airplane remains in the waiting zone and tries to land in the next period of time. This air delay is specified by establishing a flow on arc $(l_1,5),(l_2,6)$. In the end, in order to guarantee that the airplane finally arrives at its destination, the flow needs to reach final destination, i.e. the airport. This final destination is shown via a square node.

**Arc Legend**
- Flight
- Delay
- Limitation
- Destination
- Extra

**Figure 1:** G-H diagram as the network flow problem
Other real problems can of course be explained by the network model. A number of them are mentioned in what follows:
A. Non-negative costs may be considered for destination and extra arcs so that excessive delay of some flights would be prevented. This can also prevent excessive delay of continuous flights. A continuous flight is the flight where the airplane has to fly at least once again. Hence, excessive delay in the first flight leads to delay in the next one.
B. Additional flight arcs with infinite capacity and non-zero cost may be added to the network so that airplane velocity increase and decrease would also be incorporated in the model.
C. The number of airplanes in the waiting zone can be limited by placing a capacity limit on the corresponding delay arc.

4. Experimental Data
Real data is required for precisely estimating the net profit in different managerial models. Three types of data are needed: flight time, capacity prediction, and delay cost.
Flight time is the most complicated data. It is necessary to know the exact takeoff and landing time so that the real delay of each airplane would be specified.
On the other hand, since it is impossible to precisely predict the capacity of next airports, a hypothetical probability distribution is built so that situations in which airport capacity decreases in a few hours could be modeled. Fog, rainfall, snowfall, or high winds may give rise to these situations. Capacity is predicted to be time-dependent to a great extent. For instance, there may be mild fog for some time, it may then intensify and finally become mild again before disappearing. Therefore, airport capacities are modeled using three-mode Markov model. Each distribution has three capacities for each period of time: low capacity, medium capacity, and high capacity.
In these distributions, it is possible for a period with medium capacity to immediately follow a period with low, medium, or high capacity. Nevertheless, scenarios and corresponding probabilities were considered in a way that time periods with similar capacities appear one after another. This means that a period with medium capacity follows a similar period as far as possible.
From among 25 airports with most delays in 1993, data relating to five airports are compared. The required data pertaining to these five airports can be seen in Table 2 [1].

<table>
<thead>
<tr>
<th>Airport</th>
<th>Date</th>
<th>Arrivals First</th>
<th>Total</th>
<th>Maximum</th>
<th>Medium (Values)</th>
<th>Capacity Medium</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATL</td>
<td>Tuesday</td>
<td>19:01</td>
<td>138</td>
<td>19</td>
<td>3 / 94</td>
<td>9.12 .8</td>
</tr>
<tr>
<td>DCA</td>
<td>Wednesday</td>
<td>12:02</td>
<td>159</td>
<td>9</td>
<td>2 / 65</td>
<td>8.5 .3</td>
</tr>
<tr>
<td>DEN</td>
<td>Tuesday</td>
<td>15:34</td>
<td>151</td>
<td>14</td>
<td>3 / 78</td>
<td>14 .10 .6</td>
</tr>
<tr>
<td>MCO</td>
<td>Friday</td>
<td>16:11</td>
<td>77</td>
<td>8</td>
<td>1 / 64</td>
<td>4.8 .6</td>
</tr>
<tr>
<td>SEA</td>
<td>Thursday</td>
<td>16:31</td>
<td>83</td>
<td>6</td>
<td>1 / 80</td>
<td>6.4 .2</td>
</tr>
</tbody>
</table>

Each time period corresponds to a 15-minute period. Thus, in this study, capacity, delay cost, and flight time data are required for each 15-minute interval. Three types of delays are considered in this study: ground delays, air delays in the waiting zone, and air delays en route. Air delays en route were considered in locations 30 minutes away from the waiting zone and as mentioned before, the airplane adjusts its velocity in this zone in such a way that it would not have to remain in the waiting zone line for a long time before landing. Finally, ground and air delay costs were estimated to be 20.35 and 45.85 per minute,
respectively. In this study, the costs of air delays in the waiting zone and delays en route were considered to be identical. For higher efficiency, air delay cost in the waiting zone can be increased or an upper limit may be considered for the number of airplanes in the waiting zone. It needs to be mentioned that the five airports under study were selected in a way that they encompass all possible modes as far as possible. For instance, ATL, DEN, and MCO are central airports and SEA and DCA airports are semi-central. Furthermore, DCA and SEA are airports through which most of short and long flights take place, respectively. Other airports include a combination of both types of flights. As Table 2 shows, the experiment was conducted in the afternoon so that the problem scale would be relatively small. In addition, the experiment is conducted in the middle of the week (for MCO, DEN, and ATL) and on the ending days of the week (for DCA and SEA). Since there are fewer flights on the weekend (Saturdays and Sundays), no experiment was conducted on these days [1].

5. Empirical Analysis of the Model

There are two parts for analysis: answer generation and simulation. In the answer generation stage, data pertaining to the five airports were used as the inputs to the three mentioned managerial models. In NFC and FCFS definite models, the average of capacities was used. In MSDNF random model, numerous hypothetical samples for capacity values were used. Once answers were obtained from different models, statistical simulation was used for estimating the real accuracy of answers. The reason for using simulation was that different managerial models find an optimum answer for a simplified form of the problem that may not be an answer with the least cost for the main problem. A hypothetical capacity scenario was generated in each stage of the simulation, then the scenario in the model that conformed the most with the simulated scenario was obtained. This conformity is based on the primary periods of time. In fact, the best conformity belongs to the scenario that has the most in common with the simulated scenario in more primary periods of time. This new generated scenario was used for estimating the number of airplanes that reach the waiting zone in each period of time. Then, decisions were made accordingly with regard to air delay, ground delay, and air delay en route allocated to each flight. Results of this experiment may be seen in Table 3 [1].

Three types of MSDNF models with different numbers of scenarios were investigated in this experiment, including:

a. 150-SDNF: 150-scenario dynamic network flow
b. 100-SDNF: 100-scenario dynamic network flow
c. 50-SDNF: 50-scenario dynamic network flow

Also, the last column in Table 3 shows a 95-percent safety interval from the society mean obtained from the central limit theorem. Results obtained from Table 3 are described here:

A. ATL Airport

For this airport, MSDNF model leads to a considerable economization in cost compared with NFC and FCFS. Additionally, it causes a great reduction in standard deviation. For this central airport in which numerous flights take place, MSDNF model has the least average cost.

B. DCA Airport

Contrary to expectations, NFC model has a lower cost for this airport compared with MSDNF and FCFS models. However, the difference is not very huge, because NFC has a better performance compared with MSDNF as much as 2 percent. Moreover, MSDNF has a lower standard deviation. It should of course be noted that NFC considers no ground delay, whereas MSDNF allocates considerable ground delays to flights.

C. DEN Airport

Using 150-scenario MSDNF model had the best efficiency in this airport. As Table 3 demonstrates, the average cost corresponding to 50-SDNF and FCFS models is 30497 and 30422 dollars, respectively, which is not that much different. This confirms the fact that increase in the number of scenarios raises model efficiency. However, the standard deviation of all MSDNF models is lower than that of NFC and FCFS.

D. MCO Airport

Results pertaining to this airport fully meet the expectations, in a way that using NFC model had the highest average cost and using MSDNF model had the lowest average cost. Even the 50-scenario MSDNF model was more efficient than NFC and FCFS models.

E. SEA Airport

It was expected in this airport, where short flights mostly take place, that MSDNF model be more efficient than NFC model. Results are in accordance with this fact. Although SEA is not a central airport, delay costs are relatively high.
CONCLUSION
On the whole, the average efficiency of multiple-scenario models was better than other models. In addition, increase in the number of scenarios in MSDNF model reduced both the average cost and variance.

Table 3: Simulated delay costs for different airports

<table>
<thead>
<tr>
<th>Airport</th>
<th>Model</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Average</th>
<th>Standard deviation</th>
<th>Safety interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATL</td>
<td>100-SDNF</td>
<td>3968</td>
<td>113438</td>
<td>35028</td>
<td>19830</td>
<td>34905–35151</td>
</tr>
<tr>
<td></td>
<td>50-SDNF</td>
<td>5189</td>
<td>115755</td>
<td>35969</td>
<td>20660</td>
<td>35840–36097</td>
</tr>
<tr>
<td></td>
<td>FCFS</td>
<td>9158</td>
<td>139831</td>
<td>43938</td>
<td>28567</td>
<td>28567–43761</td>
</tr>
<tr>
<td></td>
<td>NFC</td>
<td>0</td>
<td>151305</td>
<td>51891</td>
<td>31475</td>
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The relative profit of MSDNF model compared with other models is shown in Table 4 [1]. These results indicate that the multiple-scenario dynamic network flow model improves air traffic delay cost compared with the current first-come first-served policy as much as 9.1%.

Table 4: Relative profit of the multiple-scenario model

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REFERENCES

**Citation of This Article**